POPULATION ECONOMICS

Christoph Borgmann

Social Security, Demographics, and Risk

Springer

Population Economics

Titles in the Series

Jacques J. Siegers · Jenny de Jong-Gierveld · Evert van Imhoff (Eds.)
Female Labour Market Behaviour and Fertility

Henrik P. Van Dalen
Economic Policy in a Demographically Divided World

Dieter Bös · Sijbren Cnossen (Eds.)
Fiscal Implications of an Aging Population

Klaus F. Zimmermann (Ed.)
Migration and Economic Development

Nico Heerink
Population Growth, Income Distribution, and Economic Development
(out of print)

Tommy Bengtsson (Ed.)
Population, Economy, and Welfare in Sweden

J. Haisken-De New
Migration and the Inter-Industry Wage Structure in Germany

Dennis A. Ahlburg · Allen C. Kelly · K. Oppenheim Mason (Eds.)
The Impact of Population Growth on Well-being
in Developing Countries

Holger Bonin
Generational Accounting

Klaus F. Zimmermann · Michael Vogler (Eds.)
Family, Household and Work

Klaus F. Zimmermann · Amelie Constant (Eds.)
How Labor Migrants Fare

Christoph Borgmann

Social Security, Demographics, and Risk

With 20 Figures
and 18 Tables

Springer

Dr. Christoph Borgmann
Blumenrain 29
CH-8702 Zollikon
Switzerland
E-mail: Christoph_Borgmann@McKinsey.com

Printed with the support of the VG Wort,
Förderungs- und Beihilfefonds Wissenschaft

Cataloging-in-Publication Data applied for

A catalog record for this book is available from the Library of Congress.

Bibliographic information published by Die Deutsche Bibliothek
Die Deutsche Bibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data available in the internet at *http://dnb.ddb.de*

ISBN 3-540-22268-5 Springer Berlin Heidelberg New York

Springer is a part of Springer Science+Business Media
springeronline.com

Printed in Germany

Cover design: Erich Kirchner, Heidelberg
Production: Helmut Petri
Typesetting: Steingraeber
Printing: betz-druck

SPIN 11015949 Printed on acid-free paper – 42/3130 – 5 4 3 2 1 0

In memory of Jürgen W. Borgmann

Preface

This study has been written during my time at the Institute of Public Finance at the Albert–Ludwigs–University of Freiburg, Germany. I am indebted to a number of people who have made this work possible. First and foremost, I thank my teacher Prof. Dr. Bernd Raffelhüschen with whom I had the pleasure of working for many exciting years. The German term “Doktorvater” alludes to the fatherly role of a thesis supervisor, and he has truely lived up to this role. Also, I am grateful to Prof. Dr. Dr. h.c. Hans-Hermann Francke for his co-correction and the valuable comments he has given me. In addition, I wish to thank my family. Especially, my wife, Daniela, has not only supported me with her love and affection but has also kept an eye on my discipline when the going got tough. The greatest sacrifice in the course of working on this thesis, have been the miles and many hours away from her. Unfortunately, the years of my doctoral studies were overshadowed by the death of my father. To him and my mother, Eva-Maria Borgmann, I am immensely grateful for their love and support throughout my life. Daniel Besendorfer and Erik Lüth have been close companions and friends at the Institute of Public Finance from my first days in Freiburg, and I have gained from their friendship enormously. This work has largely profited from comments and suggestions of many colleagues who have also been great fun to work with. Among them, I especially thank Holger Bonin, Oliver Ehrentraut, Matthias Heidler, Stefan Fetzer, Pascal Krimmer, and Stefan Moog. I gratefully acknowledge financial support provided by the Friedrich-Naumann-Foundation during the first year of my doctoral studies. Also, I thank Phillipe Bacchetta and the faculty and staff of the Study Center Gerzensee to provide me with the opportunity to study the intriguing depth of economics in a unique setting overlooking the Swiss Alps. Finally, I thank my flat-mate Dietmar Eifert and my dear friend Patrick Neise for helping me to relax and forget the issues of aging and social security occasionally.

Contents

List of Symbols

A_t	technology in the macroeconomic production function
ADJ	adjustment factor for pension benefits under early retirement options
α	capital share / feedback effect of demographic changes on wage rate
β	per-capita benefits from the public pension program to the members of the old generation
B_t	government debt in period t
b_t	government debt per old capita in period t
c_t^y	per-capita consumption of the young generation at date t
c_t^o	per-capita consumption of the old generation at date t
γ	fixed contribution rate to the social security scheme
δ	discount factor for time preference
D^c	date to which benefits are discounted for cohort c
Def_t	deficit of the public sector in period t
$\mathrm{E}[\cdot]$	expectation operator
$\hat{\varepsilon}, \hat{\epsilon}$	labor income shock
ζ	Bismarckian factor for the public pension program
θ	policy parameter for longevity-indexation of social security benefits
K	aggregate capital stock
k	capital stock per working capita
μ^z	expected excess-return of risky asset z over a risk-less asset: $\mu^z \equiv \mathrm{E}\big[\log[1+z] - \log[1+r^f]\big]$
ξ_{xy}	elasticity of x with respect to y
n	rate of population growth (deterministic)
$\hat{\eta}$	stochastic component of the rate of population growth
N_t, N_t^y	population size of the young cohort at time t
N_t^m	population size of the middle-aged cohort at time t (in 3 gen. OLG)
N_t^o	population size of the old cohort at time t
p	survival probability (deterministic), $p \in [0,1]$
$\hat{\pi}$	stochastic survival probability $\hat{=}$ longevity risk
r_t	interest rate at time t
R_t	gross interest rate: $R \equiv 1 + r$
r^f	risk free interest rate
R_t^k	gross return on risky capital: $R_t^k \equiv v_t + r_t$
ρ, ϱ	policy parameter for demographic indexation of social security benefits

s_t^y	per-capita savings of the young generation at date t
σ_x^2	variance of random variable x
ς	coefficient of relative risk aversion
v_t	value of old capital in period t
w_t	wage rate / per-capita labor income at time t
y_t	present value of per-capita life-cycle resources of the generation born in t
τ_t^y	contribution to the social security system by the young generation (lump-sum or tax rate)
T_t	aggregate tax receipts from lump-sum tax on the old: $T_t = \phi_t^o N_t^o$
ϕ_t^o	lump-sum tax on members of the old generation in period t
φ	fixed replacement rate in inflation-indexed social security scheme
ψ	replacement rate for wage-indexed defined benefit social security scheme
ω	portfolio weight
ω^p	proportion of financial assets that are invested in risky assets
ω^e	proportion of total assets that are invested in risky assets
ω^b	proportion of total assets that are invested in bonds
ω^{ss}	proportion of total assets invested in social security

List of Abbreviations

Abbreviations (without German benefit formula)

AIR	assumed investment rate of return (for variable annuities)
CAPM	capital asset pricing model
CPI	consumer price index
CRRA	constant relative risk aversion
DC	pension policy: wage-indexation with defined contributions
DB	pension policy: wage-indexation with defined benefits
EARLY	scenario with early retirement in Chapter 6
EARLY ED	scenario with early retirement and education in Chapter 6
EVS	German consumer and expenditure survey
GRV	German public pension program
LHS	left hand side
n.i.i.d.	normal independent identically distributed
NPV	net present value
OAD	old-age dependency ratio
OECD	Organization for Economic Cooperation and Developement
OLG	overlapping-generations
PIT	present value of the perpetual implicit tax of social security
PAYG	pay-as-you-go
RGC	measure of relative generosity of the GRV for single cohorts
RGT	measure of relative generosity of the GRV over time

$RRG92$	pension reform act of 1992
$RRG99$	pension reform act of 1999
RHS	right hand side
SSREV	social security reserves
SSW	social security wealth
STAN	scenario for the standard retiree in Chapter 6
TFP	total factor productivity
UNDP	United Nations Population Division

Abbreviations for the German benefit formula

AB	statistical measure of average earnings of employed during past years
AR	statistical measure of value of entitlement
AVA	share of gross earnings for savings in individual accounts (4% in 2009)
$AVmWG$	pension reform act legislated in 2001
BE	average gross earnings of employees and workers
DF	demographic factor
EP	years of coverage scaled by average earnings
L_t^{65}	remaining life-expectancy of 65 year old in year t
NQ	quota of net–to-gross-earnings (including employees share of contributions to all mandatory insurances)
$pVhs$	personal earnings relative to average earnings
RA	retirement age
RAS	adjustment of benefits that are paid to retirees who are already retired
RF	adjustment factor depending on type of pension
RQ	quota of net-to-gross-pensions
RVB	contributions to GRV (employers and employees share)
St	scaling of benefits depending on type of pension
Vj	years of coverage
ZF	adjustment factor for early retirement

Chapter 1
Introduction

During the past two decades, economists and politicians have increasingly discussed how to sustain pay-as-you-go public pension schemes during the demographic transition that is currently taking place in the aging societies of the developed world. As a result one can notice a shift from mainly pay-as-you-go financed pension schemes towards systems with a stronger weight on funded components for old-age provision. With this shift from unfunded to funded financing of the social security system, the question of risk aspects of the different pension systems has moved into focus of the research agenda on social security. In this book, we first provide an overview of the central issue of social security and will then take a close look at social security under uncertainty with a focus on the influence of demographics on systems of old-age provision.

Basically, there are three important directions of arguments in the debate on social security. The first of the three directions is steered towards the *fundamentals of social security* in a deterministic world. By this we mean the fundamental differences between a funded system of old-age provision and a pay-as-you-go financed social security program from a macroeconomic perspective under certainty. The single most important fundamental insight concerns the difference in returns of the two respective systems: while savings in a funded system yield a return equal to the interest rate, pay-as-you-go social security has an implicit return of the size of the growth rate of the economy. This fact was first recognized by Henry Aaron in 1966.

The second direction of the discussion focuses on the issue of *reforming social security during demographic transition*, where demographic transition refers to the phenomena of aging societies due to low fertility and reduced mortality. Since we know from the "fundamentals" that there is a close connection between the implicit return of social security and the population growth rate, it is obvious that social security is not a favorable investment vehicle during demographic transition. Thus, the political debate on social security and large parts of the academic literature have addressed the question of if and how social security should be reformed in the light of these demographic developments.

Finally, and of special relevance to this book, the third string of arguments concentrates on *social security and issues of risk, risk sharing, and risk diversification*. With pension reforms shifting public pension schemes towards systems with a larger degree of funded elements, the understanding of the influence of macroeconomic uncertainty on old-age provision under different types of public pension systems has gained increasing importance. Generally, under uncertainty social security can act as an instrument to generate intergenerational risk sharing when market solutions do not provide such insurance.

Following the three general directions in the discussion on social security, this volume is structured in three parts: the first part (Chap. 2) covers the “fundamentals” and the second part (Chap. 3) surveys the discussion on social security and its reform during demographic transition. Finally, the third and largest part (Chaps. 4 through 7) analyzes various aspects of risk for old-age provision with a focus on the role of social security as an institution for intergenerational risk sharing and risk diversification. We will now introduce these three parts in more detail.

In Chap. 2, we present a dense summary of the most fundamental results on the issue of pay-as-you-go social security. They are presented in a simple and insightful analytical framework by working from the budget constraint of the household. The results presented in detail are a representation of the Aaron-Theorem, the equivalence of public debt and social security, and a proof that under certain conditions, the burden of social security on the current young and all future generations equals the benefit payments to the current old. We discuss under which conditions the “ghost of dynamic inefficiency” may actually provide an argument in favor of social security. Additionally, issues in the specific design of pay-as-you-go social security are addressed. These include the indexation of social security, labor supply distortions due to generous early retirement options in public pension programs, and redistributive elements in social security. While none of the arguments presented in this part are genuinely new, the presentation of these results nevertheless constitutes an addition to the already existing literature for at least two reasons: first, the extensive and consistent coverage of these issues, including the formal representations on such a limited space, is – to the author’s knowledge – unique. Second, the proof for the equivalence of social security and public debt in a deterministic closed economy is more general than proofs of this point in the earlier literature.

The second part addresses social security during demographic transition. In Chap. 3, we survey the literature covering the issues of demographic transition, pensions, and pension reform. To give a clear picture of what we mean by demographic transition, we start out with a presentation of the demographic “facts”: the demographic developments throughout the world from the year 1950 until today and the projection of future developments in the age structure until the year 2050 are reported, according to the population projection of the United Nations. We then proceed to discuss the impact of the demographic transition on fiscal sustainability. To conclude this part, we summarize the findings on aging, factor prices, and pension reform derived from studies applying computable, dynamic, general equilibrium models. From a macroeconomic perspective, one can anticipate that the process of population aging will have an influence on the productive capacities of the economy. Labor will become increasingly scarce while capital will be relative abundant. A rise in the wage rate and a decrease in the interest rate should be expected. In addition, current pension reforms and the prospect of future pension reform may further increase the changes of wage and interest rates. On the other hand, international capital flows may prevent large movements in factor prices. Importantly, the different generations alive during the aging process are not affected equally by these changes. A welfare analysis of pension reform needs to consider the influences of factor price changes on the utility of the different generations caused by demographic transition and by pension reforms themselves.

The third and largest part of this book addresses social security against the background of risk, risk sharing, and risk diversification. Chapter 4 is intended to serve as an introduction to this third part. For a very simple example, we show how intergenerational risk sharing via social security can serve as a substitute for the missing market of intertemporal insurance contracts. Furthermore, to give a broad understanding of risk components of old-age provision from a macroeconomic perspective, we classify risk into three categories: demographic risks, productivity and valuation risks, and political risks. Generally, risk aspects of old-age provision have added another perspective to the analysis of social security that has been largely neglected in the approaches to social security summarized in the first two parts. Robert Shiller has put this nicely in his analysis of social security as an institution for intergenerational, intragenerational, and international risk sharing (Shiller (1999b, pp.1–2)):

> But, in designing social security, the fundamental problem is the government's problem deciding on institutions on behalf of individuals, some of whom are currently capable of taking actions to manage risks and some of whom are not.

The following two chapters represent original research on very specific issues of social security, risk, and demographics. In Chap. 5, we look at the optimal design of pay-as-you-go social security when two types of risk are present: labor income risks and demographic risks. Pay-as-you-go pension programs can help to share risk amongst generations. While a wage-indexed pension program is best suited to share labor income risk, we show that the combination of stochastic labor income and stochastic population growth may reduce the possibilities for intergenerational risk sharing: in a small open economy, labor income risk can only be shared when individuals are also exposed to demographic risk. Different policies on how the demographic uncertainty is transmitted via social security are analyzed and an optimal demographic indexation is derived for a small open economy and a closed economy. A common result for all considered models and scenarios is that an indexation of the benefit formula that spreads demographic shocks across generations facilitates intergenerational risk sharing of these demographic shocks.

In Chap. 6, we address the question how the generosity of the benefit rule of the German public pension system has changed during the past three decades and how this development can be explained by demographic changes. First, we illustrate the political risk of benefit rule changes for individuals. We find that depending on the birth year and the considered scenario, the relative losses vary between 30 and nearly 60 percent. Second, we estimate how demographic developments have triggered these changes in generosity. Our results suggest that future developments of the old-age dependency ratio have an influence on the determination of generosity. The empirical evidence indicates that an implicit demographic indexation of pension benefits is already in place in Germany. However, providing specific rules for demographic indexation, as suggested in Chap. 5, can help to reduce the "political riskiness" of pension benefits for the individual.

In the last chapter, we show that social security can also serve as a tool to manage risk in terms of risk diversification. Social security is evaluated against the background of portfolio choice and financial markets. For the matter of portfolio choice, social security should be seen as a "quasi-asset" in the basket of an individual's wealth and should thus be taken into consideration in the optimal choice of portfolio-allocation. Additionally, asset allocation for old-age provision has several features that makes it unique from a standard (short term) investment decision. We also show how the government should take account of the advantages of risk diversification via pay-as-you-go social security. A portfolio approach to designing social security is presented. In the same line of argument, international diversification of the portfolio should be recognized as another possible way of spreading risk that is of special relevance in connection with portfolio choices for old-age provision. Finally, we also dwell on further issues where financial markets and social security are closely connected. More specifically, we touch on the issue of annuities as instruments in old-age provision and we show how financial markets can be used to price the cost of minimum guarantees. To conclude this chapter, we point to the future, by showing how new – currently non-existent – capital markets and financial products could help in managing risk more efficiently.

All in all we find that risk aspects are an important issue for old-age provision. Risk sharing and risk diversification are essential features of social security. A broad view of risk and of the different vehicles for old-age provision should be taken. Also, the simple but very important fundamental principle of insurance applies: risk should be spread in the broadest way possible. While this principle is simple, its implementation for old-age provision may require innovative ideas. These may take the form of innovative government debt, new financial markets, or, as proposed here, indexing social security to future uncertain outcomes, e.g. demographic developments. While indexing social security does not offer the flexibility and theoretical appeal of other laissez faire solutions (e.g. the creation of a new financial order), expanding social security by demographic indexation is simple to put into practice, and in addition, it would eliminate some of the "political riskiness" underlying the public pension program.

Chapter 2
Fundamental Results on Social Security in a Deterministic Economy

In this chapter, we address fundamental issues of social security from a macroeconomic perspective.[1] The analysis focuses on a simple and consistent presentation of the most important results on social security. This is done in an analytic yet easy to follow framework. In most cases the results can be shown just by comparing the life-cycle budget constraints of the individual household under the different policies. For some results it is necessary to consider the general equilibrium effects, but this is done in a quite general framework with only the very basic assumptions on household behavior, market efficiency, and production technology. All in all, this chapter assesses the basic results of social security in a deterministic world in one comprehensive framework focusing on the economic basics. At the same time, the underlying assumptions and their implications are always discussed. The single arguments are thereby put into a broader perspective. Institutional details of specific countries are only of secondary relevance. A general typology of social security is nevertheless presented.

The analytical framework is a two period, overlapping generations economy in three settings: an economy without government activity, an economy with pay-as-you-go (PAYG) social security, and an economy with public debt. In Sect. 2.1, the government absent economy is compared with the social security case. In Sect. 2.2, we show the parallels of social security and public debt. In Sect. 2.3, we present a straight forward proof that the present value of the perpetual implicit tax of social security equals the current benefit payments. However, some conclusions are only valid in the case of a small open economy and in dynamic efficient steady states. The importance and relevance of these assumptions – especially the dynamic efficiency assumption – are discussed, and we point to the possible welfare increasing role that asset bubbles can have in the dynamic inefficient region in place of social security or public debt. The problem of switching regimes is also touched upon. In Sect. 2.4 we look at various issues of the specific design of social security. Uncertainty and the role of social security to serve as a tool for intergenerational risk sharing will only be introduced in Chap. 4.

[1]This Chapter draws on previous work by Borgmann (2001). See also Diamond (1997), Sinn (2000), and Feldstein and Liebman (2001) for surveys on social security.

2.1 PAYG versus Funded Social Security under Certainty

2.1.1 The Framework

The standard framework to investigate questions concerning social security is a model of overlapping generations (OLG) as put forward by Samuelson (1958) and Diamond (1965). In order to reduce the complexity, we consider an economy in which one generation consists of N_t^y individuals who are working in period t and are retired in period $t+1$. So in every period there are two cohorts alive: the young, who represent the current working population and the old, who represent the current retired part of the population. The growth rate of the population is denoted n_t, and so the demographic process is described by:

$$N_t^y = N_{t-1}^y(1+n_{t-1}). \tag{2.1}$$

2.1.2 The Budget Constraint Under a Funded System

The representative young agent works a fixed number of hours, which we scale to unity for simplicity, and earns wage income w_t^y. This labor income is divided between consumption when young (c_t^y) and savings for retirement (s_t^y).

$$c_t^y = w_t^y - s_t^y \tag{2.2}$$

These savings will be invested in capital. When old, this agent will want to consume the return of his savings (initial investment plus interest minus depreciation). Since by assumption, the individual does not work when old, the return of his savings will equal his entire consumption when old.

$$c_{t+1}^o = (1+r_{t+1})s_t^y \tag{2.3}$$

Solving the second period budget constraint for s_t^y and substituting into the first period budget constraint produces the life-cycle budget constraint of the representative household:

$$c_t^y + \frac{c_{t+1}^o}{1+r_{t+1}} = w_t^y. \tag{2.4}$$

This represents the very simplest setup of an overlapping generations economy without government activity. In a next step we will introduce a tax-transfer system that taxes the young and provides a transfer to the old. Such a scheme corresponds to the basic setup of pay-as-you-go organized social security. We will then distinguish the two schemes by comparing the respective life-cycle budget constraint of the representative households under the alternative schemes.

2.1.3 The Budget Constraint Under PAYG Social Security

We now introduce a mandatory PAYG system. The government, in form of a social security authority, collects lump-sum taxes τ^y from the working population and distributes transfers β^o to the old individuals. In a matured[2] system, the period budget constraints equivalent to equations (2.2) and (2.3) are then:[3]

$$c^{y,ss} = w_t^y - s_t^{y,ss} - \tau_t^y, \text{ and} \tag{2.5}$$

$$c_{t+1}^{o,ss} = (1 + r_{t+1})s_t^{y,ss} + \beta_{t+1}^o. \tag{2.6}$$

By combining the two periods budget constraints as before, we get the life-cycle budget constraint of the household in an economy with mandatory social security:

$$c_t^y + \frac{c_{t+1}^o}{1 + r_{t+1}} = w_t^y - \tau_t^y + \frac{\beta_{t+1}^o}{1 + r_{t+1}}. \tag{2.7}$$

For simplicity, let us consider a social security policy where the authority is not allowed to run any debt and has to guarantee a fixed contribution rate τ for all times. Under such a policy, all transfers to the old β_t^o are always covered by the tax payments of the current young. The budget constraint of the social security is therefore $\beta_t^o N_t^o = \tau^y N_t^y$. Since the old generation in t is equal to the young generation in $t-1$ and so $\frac{N_t^y}{N_t^o} = \frac{N_t^y}{N_{t-1}^y} = 1 + n_{t-1}$, we can write the social security budget constraint in per capita terms:

$$\beta_t^o = \tau(1 + n_{t-1}). \tag{2.8}$$

Using the social security budget constraint given in Eq. (2.8), the life-cycle budget constraint of the individuals (Eq. (2.7)) can be rewritten as:

$$c_t^y + \frac{c_{t+1}^o}{1 + r_{t+1}} = w_t^y - \tau^y \cdot \frac{r_{t+1} - n_t}{1 + r_{t+1}}. \tag{2.9}$$

This is a version of Aaron's well-known equation (see Aaron (1966)), that states that the yield of a PAYG social security system equals the growth rate of the population[4], while the return of a funded system equals the interest rate which, in turn,

[2] A system is matured when the impact of the social security scheme on the life-cycle budget constraint is identical for all living generations. In this case, the system is matured after one period when the old generation receiving benefits has already paid contributions. In reality, even minor changes of social security can often take over 60 years to mature.

[3] For an easier distinction between the various schemes, the superscript ss is adopted for social security. We will later use superscript pd for variables concerning the public debt scheme.

[4] We abstract from productivity growth here. Therefore the yield of PAYG social security is only the growth rate of the population n. Depending on the implementation of the social security scheme, the benefits of technological progress can or cannot be shared with the old population. In the simplest case, the old generation benefits completely from technological progress and in the setting of the neoclassical growth model, one could define $1 + g \equiv (1 + n)(1 + x) \approx 1 + n + x$ where x is the technological progress. The yield of the PAYG system would then be the total growth rate of the economy g.

should equal the marginal product of capital. This is exactly what can be seen from comparing equations (2.9) and (2.4): the effect on the individual life-cycle resources of a forced participation in the social security scheme, equal to $-\tau_t^y \frac{(r_{t+1}-n_t)}{1+r_{t+1}}$, since τ is the amount of resources that have to be contributed to the social security system and $r - n$ is the difference in yields of the funded system and the obligatory PAYG system. For now we will assume that $r > n$, and so the PAYG system puts a burden on the households by reducing the life-cycle resources. The difference in yields has often been called the implicit tax component of PAYG social security schemes. The economic requirements under which the assumption $r > n$ holds and the implications of this will be briefly discussed in Sect. 2.3.3. In Proposition 1, we summarize the results so far:

Proposition 1 *The difference in yields of a PAYG social security system in comparison to a funded system equals $n_t - r_{t+1}$. For $r > n$ and a compulsory lump-sum contribution τ to the PAYG social security at date t, the loss of life-cycle resources to the representative household equals $\tau \frac{r_{t+1}-n_t}{1+r_{t+1}}$ in present value terms.*

2.2 The Equivalence of Public Debt and Social Security

There have been a number of authors that have shown that public debt is equivalent to a social security scheme.[5] Here, we show this argument first in the insightful way of looking solely at the budget constraint on the individual level. In present value terms, the life-cycle budget constraint of the household will be identical given a specific policy rule.

In order to prove that this equivalence will also hold in a macroeconomic context, one has to show that, even though the saving behavior of the households differs in the two respective schemes, macroeconomic saving will nevertheless be equivalent under both systems. This is due to the government absorbing part of the households' savings in order to issue the public debt. On a macroeconomic level the net saving rate of households and the government is then equal to households' savings in the social security case.

2.2.1 An Economy with Public Debt

Consider an economy with existing public debt B_t, but no current public expenditures. Then the public deficit Def_t equals the difference between interest payments on existing debt $r_t B_t$ and tax revenue T_t. Since next period public debt is the sum of existing debt and public deficit, we can write the public sector budget constraint as $Def_t = B_{t+1} - B_t = r_t B_t - T_t$. Consider the case where the government levies

[5] Among them Raffelhüschen (1989a), Gale (1990) and Bohn (1997). Discussing the U.S. reform proposals of social security of the mid 1990s, Bohn (1997) states a number of results on neutrality of taxes and social security contributions, and of debt financed trust funds and social security.

a lump-sum tax ϕ_t^o on the old generation, so that tax revenue equals $T_t = \phi_t^o N_t^o$. If the government follows a policy rule under which the debt per old capita $b_t \equiv \frac{B_t}{N_t^o}$ is kept constant ($b_t = b_{t+i} = b\ \forall i$), the public sector budget constraint can be solved for the value of the lump-sum tax:[6]

$$\phi_t^o = (r_t - n_{t-1})b. \tag{2.10}$$

2.2.2 Equivalence in a Microeconomic Context

For comparing social security and public debt in a microeconomic context, one has to distinguish between the effect on the life-cycle resources and consumption on the one side and the reaction of savings on the other side. In this section, we will show first that for a specific size of public debt, the effects of public debt and of social security on the life-cycle resources of the households are equivalent. In the following section, we will then prove that the macroeconomic assumptions on capital stock, wage, and interest rates, necessary for the microeconomic equivalence, are in fact true.

Assuming for now that wages and interest rates are the same under both systems, it is straightforward to say that consumption when young and old will also be identical under both regimes.[7] Even though life-cycle resources and the consumption paths are equivalent, private savings will differ.

Again, we start out from the budget constraint of the households. Since no taxes have to be paid in the first period, the entire labor income is distributed between consumption and savings.

$$c_t^{y,pd} = w_t^y - s_t^{y,pd} \tag{2.11}$$

The gross return on savings will be used to finance consumption and the lump-sum tax payable when old. So the second period budget constraint equals Eq. (2.12).

$$c_{t+1}^{o,pd} = s_t^{y,pd}(1 + r_{t+1}) - \phi_{t+1} \tag{2.12}$$

Under the policy rule of constant per-capita debt as described in Eq. (2.10), one can solve equations (2.11) and (2.12) for the life-cycle budget constraint:

$$c_t^{y,pd} + \frac{c_{t+1}^{o,pd}}{1 + r_{t+1}} = w_t^y - b \cdot \frac{r_{t+1} - n_t}{1 + r_{t+1}}. \tag{2.13}$$

Comparing the life-cycle budget constraints given in equations (2.9) and (2.13) for social security and public debt, respectively, the reader will immediately note the

[6]Note that $N_{t+1}^o / N_t^o = N_t^y / N_{t-1}^y = (1 + n_{t-1})$ follows from the underlying demographic process.

[7]The timing of tax- and contribution-payments is not identical, as taxes are levied in the second period but contributions in the first period. In a more realistic setting, one would also have to assume that the households are not faced by credit constraints so that they can freely shift life-cycle resources between periods. In our example, this assumption is only necessary for the case where second period consumption is smaller than the transfers of the social security system.

similarity between the two. The two alternative regimes will exactly be equivalent for $\tau = b$. Since b is defined as debt per capita of the old, this translates into the case in which the retirement benefits to the old generation in t under social security must equal the size of next periods debt under the public debt regime in order for the two regimes to be equivalent.[8]

2.2.3 Equivalence in a Macroeconomic Context

The microeconomic equivalence will only hold if the equivalence is also true in a macroeconomic context. This is necessary, since microeconomic equivalence will only hold under identical interest and wage rates. For the factor prices (w and r) to be equal under both schemes, the per capita capital stock must be the same in both cases[9], which in turn requires macroeconomic equivalence.

To show the macroeconomic equivalence, we start out by stating the macroeconomic identities of (net)savings and investment under the alternative policies. From the respective identities one can derive the equilibrium conditions for the capital market in per capita terms.

First, we look at the economy with social security in which private savings, S^{ss} equal macroeconomic investment, I^{ss}. Setting the rate of depreciation to zero, investment equals the change in capital stock: $I_t = K_{t+1} - K_t$. Private saving is the sum of savings by young households, $S_t^{y,ss}$ and savings by old households, $S_t^{o,ss}$. The old generation will not live to see another period. Thus, it will be optimal for them not to posses any assets after this period. Obviously, the old dissave their entire savings which equals the existing capital stock K_t^{ss}. Hence, the macroeconomic identity of savings and investment can be expressed as $S_t^{y,ss} = K_{t+1}^{ss}$. As the capital-intensity k is defined per young capita, the equilibrium condition for the capital market is described by:

$$s_t^{y,ss} = k_{t+1}^{ss}(1 + n_t). \tag{2.14}$$

In an economy without social security but with public debt and lump-sum taxation of the old generation, investment I^{pd} equals the savings of private households minus newly issued public debt in the size of the deficit: $I_t^{pd} = S_t^{pd} - Def_t$. Again, investment equals the change in capital stock and private savings is the difference of the savings of the young and the dissavings of the old. Since public deficit equals the change in public debt, we have: $S_t^{y,pd} + S_t^{o,pd} = K_{t+1}^{pd} - K_t^{pd} + B_{t+1} - B_t$. As before, the old dissave all of their assets which equal the current stock of capital plus public debt. Solving for per capita terms, we have:[10]

[8]This point can easily be verified by multiplying $\tau = b$ by N_t^y, using $\tau_t^y N_t^y = \beta_t^o N_t^o$, and from the definition of public debt $B_{t+1} \equiv bN_{t+1}^o = bN_t^y$.

[9]The underlying assumption is a strictly concave production function.

[10]Note that for the macroeconomic context the definition of public debt per old capita is somewhat unlucky since capital is defined per young capita. This is why $(1 + n_t)$ appears only in conjunction with k_{t+1} and not with b_{t+1}. The reason for nevertheless using this definition is that the microeconomic equivalence is much more evident in equations (2.9) and (2.13) when debt is defined per old capita.

$$s_t^{y,pd} = k_{t+1}^{pd}(1 + n_t) + b_{t+1}. \tag{2.15}$$

In order to prove macroeconomic equivalence, we will first conjecture that the levels of capital-intensity are the same under both regimes $k^{pd} = k^{ss}$ and then go on to show that for $\tau = b$ the per capita capital stocks really are identical.

For $k^{pd} = k^{ss}$, it immediately follows that under both regimes, interest rates and wages are equal: $r^{pd} = r^{ss}$ and $w^{pd} = w^{ss}$. But then for $b = \tau$, the RHS of equations (2.9) and (2.13) are identical. Since income and prices are equal for both cases, the only necessary assumption is transitivity of preferences to conclude that the consumption-paths and, in particular, consumption when old, must be equal under social security and public debt: $c_{t+1}^{o,ss} = c_{t+1}^{o,pd}$. Inserting the period two budget equations given in equations (2.6) and (2.12), respectively, into the equality of period two consumption under the two schemes, we get:

$$(1 + r_{t+1})s_t^{y,ss} + \beta_{t+1} = (1 + r_{t+1})s_t^{y,pd} - \phi_{t+1}. \tag{2.16}$$

Using the already derived policy rules for contributions (Eq. (2.8)) and taxation (Eq. (2.10)), one obtains Eq. (2.17).

$$(1 + r_{t+1})(s_t^{y,pd} - s_t^{y,ss}) = \tau(1 + n_t) + (r_{t+1} - n_t)b \tag{2.17}$$

Substituting the capital market equilibrium conditions for savings given in equations (2.14) and (2.15), respectively and rearranging one can solve for:

$$k_{t+1}^{pd} = k_{t+1}^{ss} + \frac{\tau - b}{1 + r_{t+1}}. \tag{2.18}$$

It is easy to see that for $\tau = b$, capital-intensities are the same under public deficit with taxation of the old and social security.

Since both economies converge to a steady state, the steady state capital intensities and therefore labor income and interest rates are the same, and thus, we have verified the initial assumption necessary to show microeconomic equivalence. We summarize this result in Proposition 2.

Proposition 2 *An economy with a social security scheme is identical to an economy with constant public debt and taxation of the old if the volume of retirement benefits to the current old under the social security regime equals next period's debt under the public debt regime. In particular, the effects on the life-cycle budget constraints of the households, on the capital intensities, and therefore on wages and interest rates, are identical under the two schemes.*

The implications of this proposition for the political debate are far reaching and should always be taken into consideration when discussing social security reform. The most important implications that can be derived directly from Proposition 2 are: i) a higher (lower) tax on the old is fully equivalent to a cut (rise) in social security benefits and ii) a transition from social security to a funded system might be economically neutral if terminating social security is paid via issuance of government bonds.

2.3 The Present Value of the Perpetual Implicit Tax of PAYG Social Security

In Sect. 2.1, it was shown that the gap between market return and the return on contributions under PAYG social security can be interpreted as an implicit tax. It is frequently argued that the present value of all future implicit tax payments exactly equals the initial benefit payments to the old. Among others, this point has been made by Stiglitz et al. (1997) on an intuitive level. A proof can be found in Feldstein (1995) and Sinn (2000). Here, we will prove this statement in a very simple but illustrative way by arguing – as before – solely from the budget constraint of the individuals. We will also discuss the restrictions that need to apply.

2.3.1 A Simple Proof

An instructive way of proving the above statement is to restrict the analysis to a simple case: assume a small open economy with constant fertility $n_{t+i} = n, \forall i$ and constant world capital stock. The assumption of a small open economy allows us to neglect macroeconomic feedbacks of policy options on the capital intensity of the economy. Furthermore, since by assumption, world capital stock is constant over time, the interest rate will not vary: $r_{t+i} = r, \forall i$.

From Eq. (2.9) one can see that PAYG social security levies an implicit tax in the amount of $\tau \frac{r-n}{1+r}$ on each individual at birth. The present value of the perpetual implicit tax (PIT_t) at time t equals the sum of all future implicit tax payments discounted to date t:

$$PIT_t = \tau \frac{r-n}{1+r} \cdot N_t^y + \tau \frac{r-n}{1+r} \cdot \frac{N_{t+1}^y}{1+r} + \ldots = \tau \frac{r-n}{1+r} \cdot \sum_{i=0}^{\infty} \frac{N_{t+i}^y}{(1+r)^i} \quad (2.19)$$

In order to prove that the present value sum of the perpetual implicit tax will exactly equal the transfer payment to the current old generation, one has to show that $PIT_t = \beta_t^o N_t^o$ holds. This is done easily by applying the functional representation of the demographic process given in Eq. (2.1) and noting that the infinite geometric series over $q \equiv \frac{1+n}{1+r}$ can be expressed as $\sum_{i=0}^{\infty} q^i = \frac{1}{1-q}$ for $|q| < 1$. The assumption of dynamic efficiency $(r > n)$ is once again crucial, since otherwise $q \geq 1$. We can rewrite Eq. (2.19) as:

$$PIT_t = \tau \frac{r-n}{1+r} \cdot N_t^y \cdot \frac{1}{1 - \frac{1+n}{1+r}} = \tau N_t^y. \quad (2.20)$$

From the no-debt-policy-rule of the social security authority, we know that the contribution payments of the young generations equals the benefit payments to the current old generations and $PIT_t = \beta_t^o N_t^o$ is verified. So in fact, one can see by inspection that the present value of the perpetual implicit tax does not only equal the initial benefit payment at the introduction of PAYG social security but at every point in time, the transfer to the old generation of that period equals the present value of the perpetual implicit tax from that time on. We state these two results in Proposition 3.

Proposition 3 *In a small open economy, the social security benefit payments to the current old generation equal exactly the present value of the implicit tax of mandatory social security contributions paid by all future generations. In particular, the initial benefit payments to the old at introduction of the social security system equal the present value of the perpetual implicit tax payments.*

The parallels between social security and public debt are again obvious: under both policies the government lays a burden on future generations in order to give a windfall-profit to current generation(s). In the public debt case, this burden lies in the taxes that have to be paid to service the constant public debt, and in the social security case, it is the implicit tax of a lower-than-market return on contribution payments.

2.3.2 Policy Implications of the Equality of the Perpetual Implicit Tax and Benefit Payments

Proposition 3 implies that in a small open economy the debate on funded versus unfunded pension systems should focus on questions of intergenerational distribution rather than on efficiency arguments. In a world with strictly positive growth rates, it might even be argued that intergenerational welfare maximization calls for redistribution from future to current generations with the help of introducing a PAYG social security. However, two problems arise with this argument. The first lies in restricting the analysis to a small open economy and the second, in the implied optimal policy rule. We start with the latter: a one-time introduction of PAYG social security generates a windfall-profit for the old generation at that time, while it burdens all future generations. So what a benevolent planer would have to do is to design a slowly expanding social security scheme that gives every generation a small windfall-profit from expanding the transfer.[11] Of course such a policy will not be sustainable in the long run since the possibility to raise the contribution rate is bounded by wage income ($\tau \leq w$). Also, there are a number of competing concepts of intergenerational justice and an open dispute on how to discount the welfare of future generations.[12] The implications for an “optimal policy” will vary with the respective underlying concepts. Obviously such an “optimal policy” is purely academic and only feasible in a highly simplified setting that rules out any complications such as incentives to work.

Coming back to the underlying assumptions, it turns out that constant population growth and interest rate are not essential for the proof.[13] The important assumption, however, is the fixed capital stock due to the small open economy. Thereby a higher saving rate of a single country as a response to switching regime from unfunded to funded pension does not have an influence on the world capital stock, and therefore

[11] This expanding-transfer-policy is equivalent to introducing another PAYG social security on top of the existing one. An excellent exposition of such benefit-increases can be found in Stiglitz et al. (1997).

[12] For a recent survey on this topic, see Schwarze et al. (2003).

[13] In his proof for the same point, Sinn (2000) uses non-constant population growth and interest rates and only needs the assumption that an infinite sum similar to our sum over q converges.

domestic capital stock, income, interest rates, and wages are not altered by the countries choice of funding or not funding social security. Two problems arise with the assumption of a small open economy: first, the degree of integration of world capital markets remains an open question. Two empirical relationships suggest that a small open economy might not be the correct framework. Feldstein and Horioka (1980), Frankel (1991), and Taylor (1996) have found a strong correlation between national saving and national investment rates. This implies that changes in the domestic saving rate do have an influence on the domestic investment rate and therefore on national capital stock, national product, and factor prices of that country. The second empirical finding concerns the substantial "home bias" in equity ownership that can be observed, despite a large volume of cross-border capital movements. French and Poterba (1991) estimate that around ninety percent of equity assets of U.S. and Japanese investors are held in their domestic equity market. The second problem with the assumption of a fixed world capital stock is that even if capital markets were so strongly integrated that the domestic saving rate had no impact on the domestic capital intensity, this assumption would still break down if a major part of industrialized countries were thinking of privatizing social security. If a substantial number of countries were to change their social security policy, this again would have an effect on capital intensity. However, in most OECD countries, some sort of funding policy can be observed in the past years.[14]

Turning to the case of a closed economy, the present value of the perpetual implicit tax will still be equal to the benefit payments to the current old generation. The identity of the payment streams remains valid for lump-sum contributions payments and also for the case of payroll contributions if the tax rate is chosen every period such that a constant lump-sum tax is imitated. However, future welfare is not only affected by the absolute tax payment, but also by the change in production capacity. To make the point clear, we consider the following example. Assume that a benevolent-but-not-perfect-foresight-planner exists. After she has heard that this period's benefit payments will equal the present value of future generations' implicit tax payments, she finds it just to redistribute from future generations to the current generation and introduces a social security for that purpose. A couple of periods later, she will find that she has hurt future generations more than she had initially planned: because the capital intensity has decreased after the introduction of PAYG social security, per capita production in the economy is now lower than in the benchmark case of a funded pension scheme. Future generations are not only paying the implicit tax rate that is part of the initially

[14] Chile (1980), Australia (1991), Argentina (1993), and Mexico (1995) all practically replaced there PAYG system by a funded one. Most of the United Kingdom's occupational pension liabilities are already funded. In 2000, U.K. private-sector pension funds already had 600 billion pounds worth of investment; cf. Budd and Campbell (1998). According to Poterba et al. (2000), personal assets from the 401(k) pension plan in the U.S.A. will be substantially greater than social security plan wealth for persons retiring three decades from now. Germany has only very recently (January 2001) adopted a supplementary funded pillar to the existing PAYG social security. Feldstein (1998) covers countries that have already implemented a shift toward funded systems.

planned intergenerational redistribution but are also suffering from a lower life-cycle labor income (before taxes).

2.3.3 Dynamic Efficiency, Inefficiency, Social Security, Bubbles, and Land

Last but not least, we have to address the question of dynamic efficiency. The term dynamic efficiency characterizes an economy where the interest rate is greater or equal to the growth rate of the economy. If, on the other hand, the interest rate is smaller than the growth rate, one speaks of dynamic inefficiency. In such a situation it is possible to increase consumption of all generations, current living and future ones, by lowering the saving rate of the economy.[15] The theoretical backbone of dynamic efficiency and inefficiency is the neoclassical growth theory with its neoclassical production function and competitive factor markets where the interest rate is equal to the marginal product of capital. The decreasing marginal return of capital, implied by a concave production function, leads to this possible "over-saving" of the economy in the dynamic inefficient region. Abel et al. (1989) have transported the notion of dynamic efficiency into a world with uncertainty. In such a (more realistic) setting, the condition of $r > n$ is substituted by the requirement that net capital income exceeds investment. Only if this condition is met do the returns on capital contribute to consumption.

Obviously, it is of crucial importance whether an economy is on a dynamic efficient or inefficient growth path since in the case of dynamic inefficiency (efficiency) any government activity that decreases net-savings increases (decreases) the welfare of the economy.[16] Of course, both public debt and social security are devices to lower national saving and investment. Another implication for government policy in the dynamic inefficient region is that when issuing new public debt, the per capita debt can be reduced by just paying the interest rate and not actually paying back the principal debt, since the population is growing faster then the debt services.

Theoretically, an economy should only be in the dynamic inefficient region if for one reason or another markets are missing that prevent Pareto-improving trade. Such an argument has been made for the missing possibility to trade between generations over time. In the highly simplified setting of the two period OLG economy, the point becomes obvious: no two generations are both alive at two points in time. This missing market for trade between generations has been put forward first by Diamond (1965) as one of the arguments in favor of social security or public debt.

The discussion on whether economies can be or are in a steady state in the dynamic inefficient region remains mostly an empirical question. Abel et al. (1989) have presented strong empirical evidence that net capital incomes are larger than investments and therefore dynamic efficiency prevails. Still, it has been neglected in large parts of the literature on social security that even if an economy were in this dynamic

[15] For the case in which $r = n$, one speaks of the golden rule savings rate, since at that savings rate steady state consumption is maximized.

[16] Assuming that this government activity does not introduce any other distortions.

inefficient region, social security or public debt are not necessarily the imperative response. Tirole (1985) has shown that even for $r < n$ in the initial period, there exists an asymptotically bubbly equilibrium that is efficient and converges to the golden rule steady state. The argument relies on the fact that asset bubbles can be part of a rational expectations equilibrium if the value of the bubbles grows at the rate of the interest rate. A bubble exists if the price of an asset exceeds the market fundamentals. The bubble serves as a substitute to investments in productive capital, and therefore the capital intensity is reduced. Hence, interest rates will rise over time converging to the respective golden rule levels.

The advantage of such a bubbly equlibrium[17] is that it converges to the golden rule steady state by the forces of markets and rational behavior, whereas in the public debt or social security case, the government will have to have knowledge on the correct level of public debt necessary to reach the golden rule steady state. On the other hand, bubbles can also be potential sources of inefficiencies due to costly bubble creation or non-exhaustion of resources.

Important for the discussion on social security is the point that even though most economists feel that it is safe to assume that dynamic efficiency prevails – as we have done throughout this chapter – implementing a social security scheme might not even be necessarily the optimal policy even if the economy were initially in a dynamic inefficient state. Bubbles might do the job for the government.[18]

Finally, as Homburg (1991) and more recently Demange (2002) have shown, a competitive equilibrium is always efficient if a durable productive asset exists in a fixed amount.[19] In the real world such an asset would be land. The function of land is similar to that of the bubble: as land is an efficient tool to exchange goods over time, over-accumulation will not occur because households will substitute land for capital investments if the rate of return of capital falls beneath the growth rate of the economy.

2.3.4 On the Discussion of Switching Regimes

In the past three decades, economists have made a case for switching the PAYG social security to a funded or at least partially funded system. The arguments for such a switch centered around i) the implicit tax of social security captured by the Aaron-Theorem and ii) the loss in aggregate savings and therefore investment due to the nature of a PAYG system. The first problem with both arguments is that their relevance is an

[17] Note that national debt does not contain a bubble; it only acts as one. See Tirole (1985, p. 1518).

[18] However, Lang (1996) has a more negative view on asset bubbles and dynamic efficiency. He argues that fighting overaccumulation of capital with an asset bubble does not guarantee a Pareto-improvement. Also, he questions that bubbly equilibria fulfill the so called core property of the general equilibrium framework; cf. Lang (1996, pp. 34–41).

[19] Feldstein (1977) was the first to include a fixed factor in addition to reproducable capital in an OLG framework. See Hange (2003) for a intuitive discussion of how different generations are affected by the introduction of social security in an economy with land. However, uncertainty may change the results; cf. Richter (1993), Demange (2002), and Sect. 5.5.2.

empirical question. Argument i) only applies when we are in the dynamic efficient region (see discussion above). Argument ii) has been put in doubt by the so called Munnell-Feldstein controversy; cf. Feldstein (1974) and Munnell (1974). Economists have found empirical evidence for and against both arguments.[20] The major part of economists are, however, willing to assume that dynamic efficiency applies and that social security does in fact lower the national saving rate. Following this judgement, one runs immediately into a new problem: even though the new steady state of a funded system might constitute a clear welfare improvement in comparison to the current situation with an unfunded system, one has to take into account the transition period and the generations that are alive during that period.

The question of whether Pareto-superior switching strategies, i.e. strategies that will make every generation better or at least as well off as before, are possible has been at the center of the debate on funded vs. unfunded social security at the beginning of the 1990s. A recent survey of the literature can be found in Hirte (2000). A number of authors have shown that efficiency gains are possible.[21] However, in all cases the authors combine the effects of funding social security and changing the tax base or eliminating further distortions like early retirement incentives. The welfare gains from the latter policy change can, of course, always be realized separately without funding social security. Fenge (1995) has shown that a PAYG pension system with intragenerational fairness is Pareto-efficient. He argues that lump-sum taxes are political infeasible, and therefore the government can only control the net real wage rate. In this case a transition cannot be Pareto-improving. In a similar line of argument, Hirte (2000) points out that Pareto-superior switching strategies are not possible without also changing the tax base. Alternatively, he analyses the political feasibility of changing to a (partially) funded system by looking for majorities and comes to the conclusion that (in the German case) the prevailing system is preferred by the majority of voters.

2.4 The Typology of Social Security and Further Issues

Old-age income – that is income during the time period when no labor income is earned – can be classified into many different categories. As discussed above, from a macroeconomic perspective, the most fundamental difference is whether the system is funded or unfunded. Often the term “funded” is interpreted as a synonym for a voluntary system with individual accounts that bears no redistributive elements and where the savings are safe from the government. On the other hand, speaking of an “unfunded” system is commonly used to describe a government-run, mandatory pay-as-you-go scheme with inter- and intragenerational redistribution. While some of the elements are intimately connected with each other, it is far from compulsory that a pension program must exhibit all of the above mentioned features in this

[20] Compare Börsch-Supan (2000b) for a survey of recent empirical literature on the topic.
[21] See for example Breyer and Straub (1993) and Raffelhüschen (1993).

constellation. Following Homburg (1988), one can more broadly use the following five categories in order to construct a typology of a pension program: PAYG versus funded, mandatory versus voluntary, government-run versus privatized, insurance of life-length uncertainty versus not, and redistributive elements versus not.

The sum of all measures for old-age provision is often subsumed in the term "three-pillar (or multi-pillar) pension systems". According to this illustrative classification, the PAYG public pension scheme represents the first pillar. Company pension programs are identified as the second pillar for old-age income. Finally, all other individual savings are subsumed under the third pillar. The term "three-pillars" thus more or less classifies who is administrating the different sources of retirement income (government, company, and private). At the same time a "multi-pillared" system can be understood as a diversified approach to old-age income provision that stresses the importance of including different elements in the portfolio of assets and entitlement to benefits.

In the remaining part of this section, we elaborate on some of the issues that become important when one looks beyond the crude classification of PAYG versus funded pension schemes. Especially, we will touch on further issues of the specific design of a PAYG social security program, such as the indexation of benefits, the effect of public pension programs on the pre-retirement labor supply, and on the difference between employment-related versus flat-rate social security programs.

2.4.1 The Indexation of Benefits: Wage Indexation and Demographic Indexation

Within the category of PAYG social security schemes there are important differences regarding the indexation of the pensions. Within employer pension programs one generally divides between *defined contributions* and *defined benefits*; cf. Bodie et al. (1988). Within the former system the employer and/or the employee pay regular contributions into an individual account for each employee. Under the latter scheme the employee will be eligible to pension benefits that are determined via a formula that will take account of years of contribution payments and, in most cases, the salary.

Transferring this classification to public pension programs, one associates a defined contribution plan with a scheme where benefits are usually determined by the contributions paid during the course of the working period and are usually not contingent on any future states of the economy after entering retirement. A defined benefit plan, on the other hand, promises a certain level of benefits payment over the remaining life-time after retirement. This crude classification still leaves open certain questions, e.g. is the defined benefit plan promising a certain lump-sum annuity or is the defined benefit plan promising a certain replacement rate of future income levels. Likewise, it remains an open question whether defined contribution characterizes a system where contributions are determined as a fixed amount or as a fixed tax rate such that contributions are proportional to labor-income. Accordingly, the nomenclature of *fixed contribution rate* versus *fixed replacement rate* is preferred in the literature covering these issues for PAYG social security; cf. Thøgersen (1998) and Wagener (2003a,b). In addition, we will argue here that in order to better describe a PAYG pension scheme, it is helpful to categorize the indexation of the pension scheme according to two criteria: i) are benefits indexed to current labor income or just indexed to inflation, and ii) how do benefits react to demographic changes. We will thus speak of *wage indexation* and *demographic indexation* in order to characterize the indexation of social security.

Figure 2.1 helps to illustrate the distinctions between the different types of old-age income provision. The classifications in this figure are made from a macroeconomic perspective. Hence, the most important difference at the top: funded versus unfunded. Without going into detail here, it is indicated that under a funded scheme savings of the young may be larger, smaller, or equal to the dissaving of the elderly population in absolute terms. The depicted case describes a closed economy where the capital-ratio per working-aged capita is given by the savings of the prior generation. A further distinction, we include here, concerns the payout at retirement: the stock of savings at retirement may either be paid out lump-sum or annuitized. While the insurance of life-length risk is not a feature that is restricted to PAYG social security, empirical evidence has been provided that point to deficiencies within competitive markets due to adverse selection; cf. Walliser (1997,2000) and Warshawsky (1988). People that have a lower remaining life-expectation than the average retiree due to their personal medical history will most likely not chose to annuitize their wealth and vice versa. The issue of annuities will be covered in some more detail in Chap. 7.

Pension schemes that invest in public debt are included under the category of an unfunded system because of the macroeconomic implications of government debt. This may deviate from the subjective perception of an individual who invests privately into government bonds. But from a macroeconomic perspective, this type of "investment" will not build up capital and is therefore classified as unfunded.

Addressing the differences of PAYG social security, we emphasize the different possibilities of indexing such a public pension program. The specific design of social security will have a crucial influence on how the exposure to risks is distributed between cohorts. This matter will be at the heart of the discussion in Chap. 5. Here we only briefly point out that benefits may or may not be indexed to wages.[22] This is the most common distinction in the design of public pension programs in the literature. However, PAYG social security may also differentiate in respect to the demographic indexation. Therefore, we introduce a policy parameter ϱ that determines the demographic indexation of the pension program. The earlier mentioned cases of *fixed contribution rate* and *fixed replacement rate* are thus not necessarily equivalent to wage indexation and no wage indexation, respectively. The bottom line in Fig. 2.1 shows that these two policy options are special cases for specific combinations of wage indexation and demographic indexation.

2.4.2 Social Security and Incentives to Retire Early

Most public pension programs in the developed world set strong incentives to retire earlier than the regular retirement age. This is due to implicit taxes on labor income

[22] A number of European countries are currently introducing "notional" accounts, where within a PAYG system, benefits are determined using individual earning and an actuarial formula. For a discussion of notional accounts and financial stability, see Valdés-Prieto (2000).

in the years near the regular retirement age. The implicit tax takes the form of non-actuarially fair adjustment factors of pension benefits for individuals who are retiring

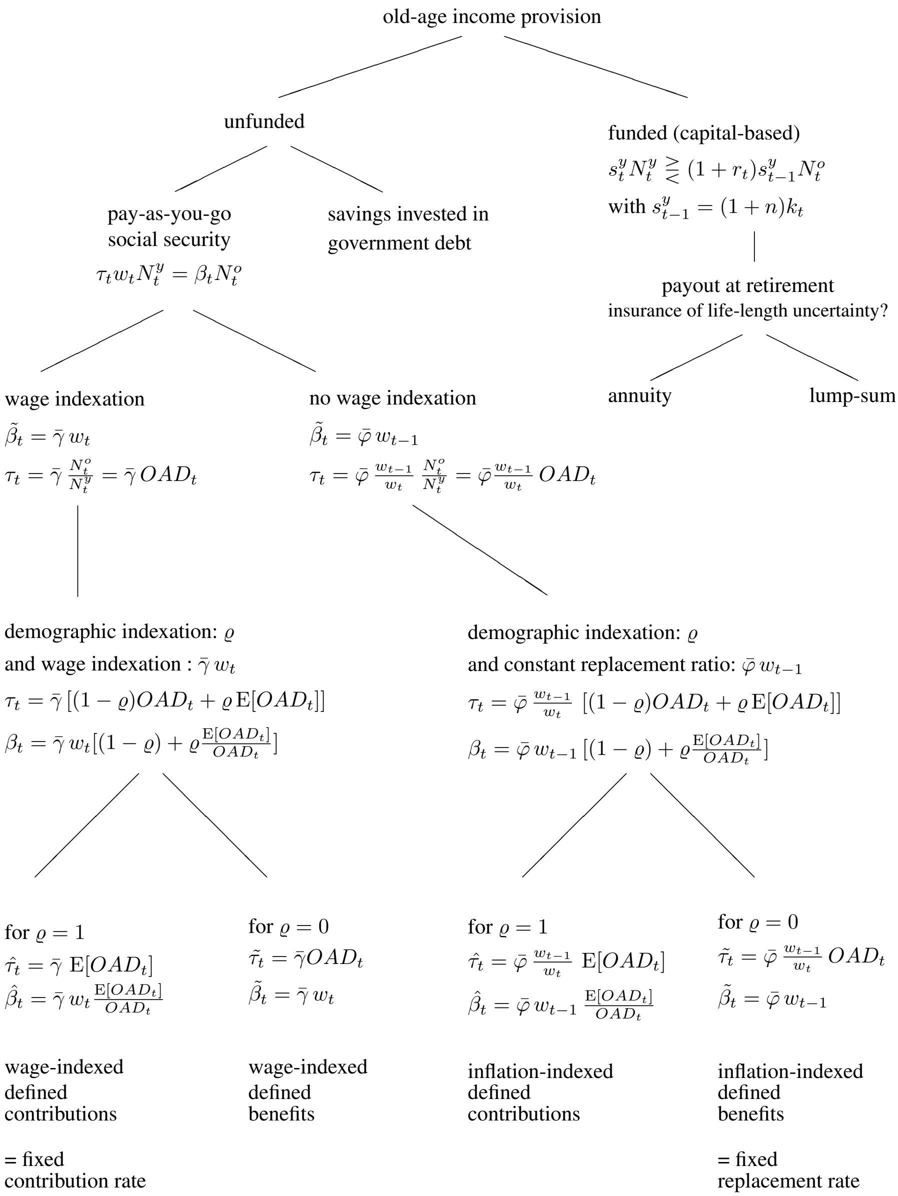


Fig. 2.1. Macroeconomic classification of old-age provision

at ages other than the regular retirement age.[23] In most developed countries, this regular retirement age is currently 65; cf. SSA(1999,2002a,b) and Table 2.2.[24]

To be precise, benefits are usually calculated on the basis that the pensioners are retiring at a given age, i.e. the regular retirement age. If the possibility of receiving benefits prior to this regular retirement age exists, one speaks of early retirement. The early pensionable age for 44 European countries is given in Table 2.2. The listed values understate the possibilities for early retirement because early retirement options for specific groups, such as long-term contribution payers, are not included. Note that the important feature is the eligibility to pension benefits prior to the regular retirement age and not the exit of the labor force. Of course, usually the pension program demands that the exit of the labor force is a necessary condition in order to be eligible. If an individual is making use of the early retirement option, an actuarially fair contract would reduce the benefit payments to this individual in comparison to an individual who is retiring at the regular retirement age because of three reasons. Firstly, the individuals are contributing for a shorter time period. Secondly, the period of retirement and therefore the duration of receiving benefits increases by early retirement.[25] And thirdly, the individual is benefiting form a "present value effect". This effect is due to the fact that the benefit stream starts earlier in time, or put the other way around: the compounded interest on the stock of entitlement should be smaller.[26]

An implicit tax occurs when the public pension program is not taking full account of all three of these effects. To be precise, a pension program can be considered as actuarially fair if the net present value of prospective pension benefits minus future contribution payments is kept equal for all retirement ages (cf. Antolin and Scarpetta (1998)). In order to calculate the non-distorting adjustment factors, one needs to calculate the net present value of the pension program for the regular retirement age, e.g. 65, and then introduce adjustment factors that will guarantee that the net present value of retiring at an age other than 65 still yields the same net present value. The net present value (NPV_i) of pension benefits for retirement at age i is given by:

[23] Non-actuarially adjustment factors increase the effective tax rate on labor income. Thus, the implicit tax rate we are speaking of here comes in addition to the implicit tax of social security contributions for all ages.

[24] Some countries are thinking about increasing this age or have already legislated such measures, e.g. the United States are gradually increasing the regular retirement from 65 today to 67 in 2027.

[25] The probability of actually receiving benefits also becomes larger because a certain fraction of the population dies in the interval between early retirement age and regular retirement age.

[26] The notion of compounded interest is a bit strange when discussing PAYG social security. An actuarially fair contract would, however, take this into consideration.

$$NPV(i) = \sum_{j=i}^{\infty} ADJ(i) \cdot \beta(65) \cdot \pi_i \cdot R^{-(j-60)} - \sum_{j=60}^{i-1} \tau \cdot w \cdot R^{-(i-1-j)} \tag{2.21}$$

with	i	retirement age
	$ADJ(i)$	adjustment factor for retiring at age i
	$\beta(65)$	yearly pension payment when retiring at 65
	π_i	survival rate at the age i
	R	gross interest rate $(1+r)$
	τ	contribution rate
	w	annual earnings

In order to compute the actuarially fair adjustment factors one needs to find $ADJ^*(i)$ such that $NPV(i) = NPV(65)$ with $ADJ(65) = 1$, where we have taken 65 as the regular retirement age. The actuarially fair adjustment factors can thus easily be calculated with the help of mortality tables and an assumed interest rate.

In Table 2.1, we report the effective adjustment factors in Germany according to legislation before and after the pension reform act of 1992 (RRG92) and the non-distorting adjustment factors as calculated by Börsch-Supan (1999) and Antolin and Scarpetta (1998).[27] The example of Germany shows that the adjustment factors are usually chosen too low, and therefore the public pension program increase the effective tax rate on labor income near the end of the working life.

Because of the specific design of social security (and often also company pensions), the implicit tax on earnings in the years before the conventional retirement age sets strong incentives to retire at the age of first benefit entitlement. It is therefore not surprising that the average age of entering retirement is far below the regular retirement age.[28] In Fig. 2.2, the implicit tax rates of 22 OECD countries are depicted together with the countries' respective average age of retirement. The figure shows two things: first, the average age of retirement is beneath 64 in all but three countries, namely Iceland, Japan, and Switzerland. Second, the international evidence suggests that the implicit tax rate has an influence on the choice of the retirement age.

Since the seminal work of Stock and Wise (1990a,b), an extensive empirical research has taken place on how economic incentives influence the labour/leisure choices near the age of retirement. In the volume edited by Gruber and Wise (1999), the institutional settings and empirical details concerning the retirement decision are presented for most developed countries. A survey of 22 OECD countries can be found in Blöndal and Scarpetta (1998). A key finding of the empirical literature on this topic is that the pension scheme does in fact have an important impact on the timing of retirement. Hence, the first impression from Fig. 2.2 can be confirmed. In addition,

[27] See also Börsch-Supan (1992) for an earlier calculation of adjustment factors using older mortality tables.

[28] See Sect. 6.1.4 for a summary of early retirement options in Germany.

Table 2.1. Effective and actuarially fair adjustment factors for early retirement in Germany

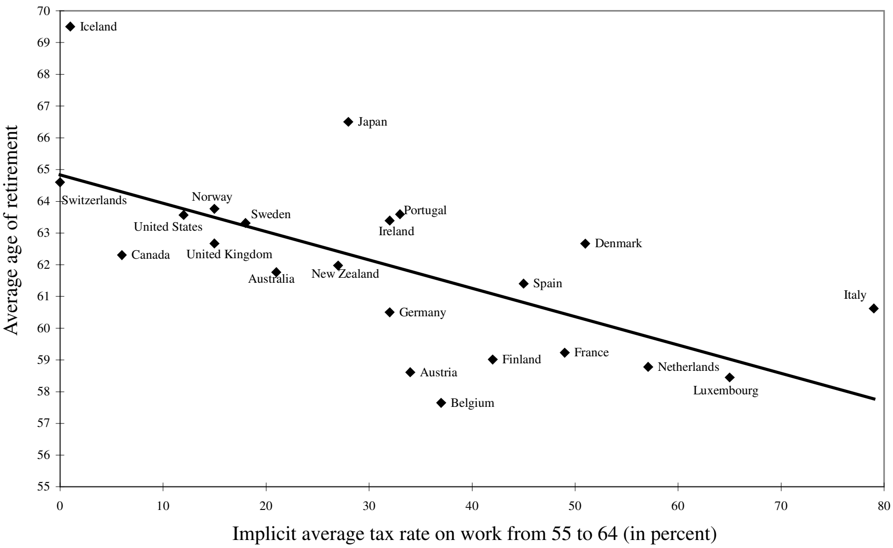
retirement age	effective pre RRG92	effective post RRG92	actuarially fair BS1999[a]	actuarially fair AS1998[b]
60	87.5	–[c]	66.0	69.1
61	90.0	–[c]	71.5	74.2
62	92.5	81.7	77.6	79.7
63	95.0	87.8	84.3	85.8
64	97.5	93.9	91.7	92.5
65	100.0	100.0	100.0	100.0
66	109.9	108.5	109.2	108.4
67	120.1	117.0	119.6	117.8
68	123.0	125.5	131.2	128.4
69	125.8	134.0	144.4	140.4
70	128.7	142.5	159.4	154.1

[a]Börsch-Supan (1999), r=3%, mortality tables 1992/94, eligibility of 40 EP at age 65.
[b]Antolin and Scarpetta (1998), r=4.5%, UN Life Tables 1985, eligibility of 45EP at age 65.
[c]Retirement not possible.

Börsch-Supan (2000a) predicts that a shift to a non-distorting pension scheme in Germany would cause the cumulative distribution of retirement at ages 59 through 61 to drop by more than ten percentage points for each respective age. Gruber and Wise (2002) survey the results of similar studies of micro estimations for twelve countries. They also come to the conclusion that while countries may differ with respect to their cultural histories and labor market institutions, the employees in all countries react similar to social security retirement incentives. Finally, simulations for theses twelve countries predict that reforms such as actuarial adjustment factors or delaying benefit entitlement will have a large effect on labor force participation for employees in the age group of 56 to 65 years.

2.4.3 Bismarckian versus Beveridgean Social Security

Public pension schemes in the developed world can be identified by two distinct models of pension provision. Pension systems of the Bismarckian type are generally aimed at maintaining a level of living standards during retirement that is comparable to the achieved level during the working life. In contrast, the aim of a Beveridgean pension scheme is poverty prevention. The two systems diverge in their basic principal: while Bismarckian social security follows the insurance principle, Beveridgean social security has its foundation in the welfare principle. According to their aims, the two philosophies of pension provisions result in very different institutional arrangements:



Source: Blöndal and Scarpetta (1998)

Note: The implicit tax rate takes into account incentives in both old-age pension and unemployment-related benefit systems.

Fig. 2.2. Implicit tax on continued work and the average age of retirement in 1995

benefits from the Bismarckian model are earnings-related, whereas the Beveridgean model is characterized by flat-rate benefits. Many of the flat-rate systems for retirement income are means-tested, i.e. eligibility depends on a persons own or family income and/or wealth. Some countries maintain a universal flat-rate scheme where benefits are paid as a uniform amount to all residents of the country (minimum years of residency is usually required), independent of prior earning histories. Universal programs usually are accompanied with a second-tier earnings-related program. Of the 44 European countries included in a survey of the Social Security Administration (cf. Table 2.2), eight countries (18%) have a flat-rate mandatory system of retirement income. In another eight countries (18%), the benefit formula contains a flat-rate component as well as earnings-related elements; 29 countries (66%) maintain a system with an earnings-related scheme, and only one country (Russia) provides universal flat-rate retirement income. Taken together, in over 80% of the European countries mandatory social security is at least partially earnings-related. Of these 44 countries only 13 countries (30%) are forcing the population to contribute to a mandatory occupational or individual retirement scheme.

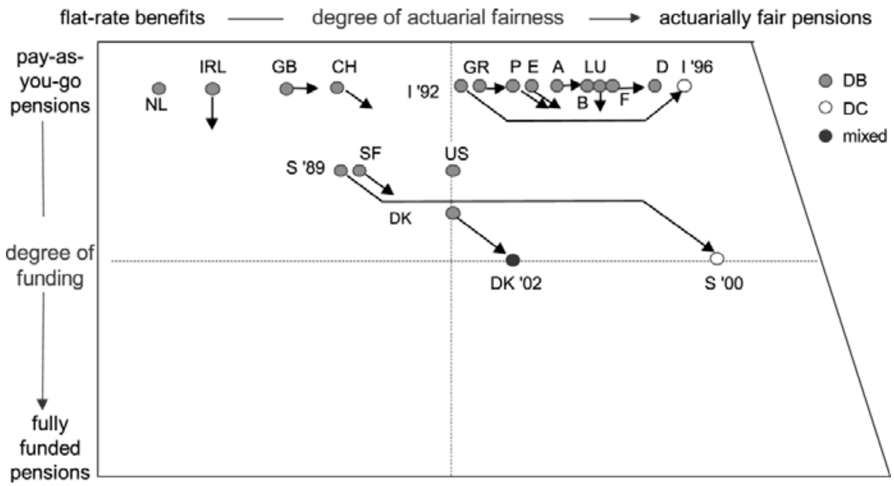
Naturally, Bismarckian social security is associated with a higher replacement rate (gross and net), but less redistribution than Beveridgean. This is also due to the fact that flat-rate programs, especially those that are means-tested or universal, are usually financed from general tax revenues. In an analytical framework, the degree of redistribution is described by making the pension benefits a function of a parameter ζ, that can be interpreted as the Bismarckian factor. Pension benefits, then, consist of two parts: a contributory part that is related to individual earnings (w) and a non-contributory part that is determined by average earnings ($\bar{w}$): $\beta_{t+1}(w) = (1+n)\gamma(\zeta w + (1-\zeta)\bar{w})$; see e.g. Casamatta et al. (2000). In reality, however, the degree of progressivity of social security is in many cases unclear even in systems that are believed to be highly redistributive. Coronado et al. (2000) and Gustman and Steinmeier (2000) both find that when taking account of lifetime-earnings and other complicating factors, such as family coverage, the U.S. public pension program may actually be regressive. The political economy of when a Bismarckian system is preferred to a Beveridgean program is dealt with in Conde-Ruiz and Profeta (2002). The advantages of providing an income maintenance at subsistence needs is not further touched upon here; cf. Sinn (1995). Instead, we will mainly concentrate on systems without intragenerational distribution in the following chapters.

2.5 Summary

This chapter provides a summary of what one should keep in mind when discussing unfunded versus funded social security. Instead of commenting on a single country's recent and current ambitions on reforming social security, we derive the most fundamental arguments in an analytical but still easy to follow way. In essence, what one should bear in mind are three arguments:

First, there is an implicit tax on social security contributions because of the lower than market internal rate of return on PAYG social security contributions. Second, depending on the exact policy, social security and public debt are equivalent or near equivalent. This is due to the fact that, under a scheme that keeps per capita public debt constant, the necessary taxes to pay interests on the existing debt equals the implicit tax of the social security contribution. Thus, funding strategies for social security that are debt financed might actually be equivalent to the initial PAYG system. Third, one has to be careful about the efficiency gains from switching from PAYG to a unfunded scheme. One gain that can always be realized (even without switching systems) is eliminating falsely set incentives concerning early retirement or an inefficient choice of the tax base. However, whether there are efficiency gains in terms of higher national product because of higher saving rates crucially depends on i) how much world capital markets are really integrated and ii) whether a single industrial country or all industrial countries are going to "privatize" social security.

Next to these fundamental arguments one has to keep in mind that the specific design of a public pension program is important, especially with regard to the rules concerning the indexation of benefits and early retirement. Table 2.2 summarizes the regulations of mandatory retirement systems across Europe. A recent survey of old-age provision, i.e. all three "pillars" of the EU-15 conutries, Switzerland, and the U.S. can be found in Fenge et al. (2003). Applying the characterization of social security as put forward by Lindbeck and Persson (2002), they visualize to what degree the social security systems in these countries are unfunded or funded and actuarially fair



Source: Fenge et al. (2003, Fig. 4.2)

Fig. 2.3. Characteristics of public pension schemes and directions of recent change, 1990–2002

or redistributive. Figure 2.3 shows the results for the first pillar, i.e. state run public pension programs. Also as a third degree of freedom, the types of indexation of the schemes are indicated. Finally, changes due to recent reforms are displayed. While the state-run schemes are often purely defined benefits schemes that are predominantly PAYG financed, expanding the focus of the analysis to all three pillars[29] shows that old-age income provision as a whole has a much higher percentage of funded components and is more often a mixture between defined benefits and defined contributions. Also, national pension schemes tend to be much less redistributive than just the public pensions schemes alone.

After this overview on social security in a deterministic world, we will turn to the process of demographic transition and its macroeconomic implication in the next chapter and will subsequently deal with questions of risk and risk sharing in the remaining chapters.

[29]Compare Fenge et al. (2003, Figure 4.3, page 133). They use the term *national* pension scheme for all three pillars. See also Werding (2003) for an English summary.

Table 2.2. Types of mandatory systems for retirement income, statutory pensionable age, and contribution rates across Europe

Country	Flat-rate	Earnings-related	Means-tested	Flat-rate universal	Occupational retirement schemes	Individual retirement schemes	Statutory pensionable age		Early pensionable age[a]		Contribution rate[b]
							Men	Women	Men	Women	
Albania	x[c]	x[c]					60	55	[d]	[d]	36[e]
Andorra		x					65	65	[d]	[d]	8
Austria		x					65	60	61.5	56.5	22.75[f]
Belarus		x	x				60	55	[d]	[d]	5.7[e]
Belgium		x	x				65	62	60	60	16.36
Bulgaria		x	x			x	61.5	56.5	[d]	[d]	31
Croatia		x				x	62	57	57	52	19.5[g]
Cyprus		x	x				65	65	[d]	[d]	12.6[ef]
Czech Republic	x[c]	x[c]					61	59	58	56	26
Denmark			x			x	67	67	60	60	[hf]
Estonia	x[c]	x[c]				x	63	58	[d]	[d]	20
Finland			x		x		65	65	60	60	21.1[f]
France		x	x		x		60	60	[d]	[d]	16.45[f]
Germany		x					65	65	[d]	[d]	19.1[f]
Greece		x					65	65	60	60	20[f]
Guernsey	x						65	65	[d]	[d]	9.9[ef]
Hungary		x				x[i]	62	62	[d]	[d]	26[ef]
Iceland			x		x		67	67	[d]	[d]	15.83
Ireland	x		x				66	66	[d]	[d]	16.75[el]
Isle of Man	x	x	x				65	60	[d]	[d]	21.9[ef]
Italy		x	x				65	60	[d]	[d]	
Jersey	x						65	65	63	63	10.5[f]

Source: SSA (2002b)

continued...

Table 2.2. (continued)

Country	Flat-rate	Earnings-related	Means-tested	Flat-rate universal	Occupational retirement schemes	Individual retirement schemes	Statutory pensionable age		Early pensionable age[a]		Contri-bution rate[b]
							Men	Women	Men	Women	
Latvia		x	x			x	61.5	58.5	60	56.5	30.86
Liechtenstein		x			x		64	62	60	60	8.8
Lithuania	x[c]	x[c]					62	58	[d]	[d]	25
Luxembourg	x[c]	x[c]					65	65	57	57	16[f]
Malta	x[j]	x					61	60	[d]	[d]	20[ef]
Moldova		x	x				62	57	[d]	[d]	30[e]
Monaco		x					65	65	60	60	28.06[ef]
Netherlands	x						65	65	[d]	[d]	28.05[f]
Norway	x	x					67	67	[d]	[d]	21.9[e]
Poland	x[c]	x[ck]				x[k]	65	60	[d]	[d]	32.52
Portugal		x	x				65	65	55	55	34.75[ef]
Romania		x					65	60	55	55	35[ef]
Russia	x[c]	x[c]		x			60	55	[d]	[d]	28[e]
San Marino		x					60	60	[d]	[d]	11.9[e]
Serbia		x					60	55	[d]	[d]	32
Slovak Republic		x	x				60	60	[d]	[d]	28[ef]
Slovenia		x					58	54	[d]	[d]	24.35[e]
Spain		x					65	65	61	61	28.3[ef]
Sweden		x	x			x	65	65	60	60	17.21[f]
Switzerland	x[c]	x[c]	x		x		65	63	[d]	[d]	23.8[f]
Ukraine		x	x				60	55	[d]	[d]	38[e]
United Kingdom	x	x	x				65	60	[d]	[d]	21.9[ef]

Source: SSA (2002b)

see next page for notes...

Footnotes and notes for Table 2.2

[a] General early pensionable age only; excludes early pensionable ages for specific groups of employees.

[b] Old-age, disability, survivors programs. Contribution rate of insured person and employer.

[c] The benefit formula contains a flat-rate component as well as an earnings-related element.

[d] The country has no early pensionable age, has one only for specific groups, or information is not available.

[e] Also includes the contribution rate for other programs.

[f] Contributions are submitted to a ceiling for some benefits.

[g] New system rates.

[h] Portion of set amount for old-age, disability, and survivors. Central and local government and other types of contributions for the other programs.

[i] Persons who became insured before June 30, 1998, or who became insured after this date but before reaching the age of 42 years, can choose between the earnings-related system or the mixed system of the earnings-related pension and private insurance.

[j] Flat-rate benefits are awarded to those who paid contributions before January 22, 1979.

[k] The old system contains a flat-rate component as well as an earnings-related element. The new system includes an earnings-related notional defined contribution (NDC) scheme and private mandatory insurance.

[l] Range according to earnings bracket. Higher rate is shown, which applies to highest earnings class.

Notes: The types of mandatory systems for retirement income are defined as follows:

Flat-rate pension: A pension of uniform amount or based on years of service or residence but independent of earnings. It is financed by payroll tax contributions from employees, employers, or both.

Earnings-related pension: A pension based on earnings. It is financed by payroll tax contributions from employees, employers, or both.

Means-tested pension: A pension paid to eligible persons whose own or family income, assets, or both fall below designated levels. It is generally financed through government contributions, with no contributions from employers or employees.

Flat-rate universal pension: A pension of uniform amount normally based on residence but independent of earnings. It is generally financed through government contributions, with no contributions from employers or employees.

Provident funds: Employee and employer contributions are set aside for each employee in publicly managed special funds. Benefits are generally paid as a lump sum with accrued interest.

Occupational retirement schemes: Employers are required by law to provide private occupational retirement schemes financed by employer and, in some cases, employee contributions. Benefits are paid as a lump sum, annuity, or pension.

Individual retirement schemes: Employees and, in some cases, employers must contribute a certain percentage of earnings to an individual account managed by a public or private fund manager chosen by the employee. The accumulated capital in the individual account is used to purchase an annuity, make programmed withdrawals, or a combination of the two and may be paid as a lump sum.

Chapter 3
Social Security and Demographic Transition

The single most important reason why issues of social security have become of such an immense interest in the past decades are the historical unique demographic changes that have taken place during the last century. The combination of low fertility rates and rapidly increasing life-expectancy have led to the phenomena of the so-called aging society. This chapter serves two goals. First, we want to illustrate how low population growth and decreasing mortality have changed the age structure of the population throughout the world. In order to do so, we report the demographic developments from 1950 until today and we look at the demographic projections according to the medium variant of the United Nations Population Division (2003a), henceforth UNDP, until 2050.[1] Second, we will survey the recent literature that has addressed the question on macroeconomic implications of an aging population. More specifically, we address the issues of fiscal sustainability and we report projected trajectories of factor prices and asset prices during demographic transition. This analysis is conducted with a specific, but not exclusive, focus on social security.

3.1 Demographic Trends Around the World

We begin in this section with presenting the demographic "facts": we show how the age structure has changed over the last fifty years and refer to the UNDP projection to show how future developments will likely look like. A second key point of the presentation here is to illustrate that there are sharp differences between the different regions of the world with respect to their demographic situation. Regional differences can be grouped according to the economic development of the respective countries. The countries that are classified as *more developed regions*[2] will all be exposed to a similar demographic trend. We also look at different subgroup of theses more developed countries, namely the OECD countries, the G7 countries, Europe, and North America to confirm that all industrialized nations are confronted by the above cited phenomena of the aging society. The *less developed regions*[3] exhibit a far more favorable age structure of the population. According to the population projections of the United

[1] The data can be downloaded from http://esa.un.org/unpp.

[2] The definitions of country-groups are given in Table B.1 in Appendix B.1.

[3] Note that the definition of *less developed regions* used here deviates from that definition of the UNDP, because we do not include the least developed countries in our definition of less

Nations Population Division (2003a), it will likely take until the year 2050 for the less developed regions to have an age structure similar to that of today's *more developed regions*. It is only the group of the *least developed regions* that is expected to still have above-replacement fertility and an old-age dependency ratio under 10 by the year 2050.

3.1.1 Fertility and Life-Expectancy: 1950 – 2050

The growth of the world's total population is determined by the number of births and deaths in every year. For a single country or region the pattern of migration must also be taken into consideration. Concentrating on the natural causes (birth and death) of population growth we will focus our discussion on two indicators that are (among others) commonly used to describe demographic processes: the total fertility rate and the life-expectancy at birth.

Fertility. Using the definition of the UNDP the total fertility rate is the "average number of children a hypothetical cohort of women would have at the end of their reproductive period if they were subject during their whole lives to the fertility rates of a given period and if they were not subject to mortality. It is expressed as children per woman."[4] Simply put, the total fertility is a measure of how many children an average woman will bear during her life. The value for the total fertility rate that will ensure the long-term replacement of the population is roughly 2.1.[5] Past values and projected future values of the total fertility rate are given for selected regions in Fig. 3.1. The fertility rates for the years 2000 through 2050 correspond to the medium-fertility assumptions that are used in the medium projection variant of the UNDP.[6] According to these assumptions, the total fertility rate converges towards a value of 1.85 by 2050 for most countries.

Figure 3.1 shows that it is mainly the *more developed regions*, especially Europe and Japan (not depicted), that have experienced below-replacement fertility since the beginning of the 1970s. North America is the region where fertility first dropped under the replacement-level. However, in North America, fertility has picked up again in the 1990s and is now roughly two. For the entire world, the total fertility rate has nearly dropped by 50 percent from a level of roughly five children per woman in 1950 to circa 2.7 today. Only the *least developed regions*, which are basically comprised of most African countries, still exhibit fertility rates far above the replacement level today. Note also that these are the only regions were fertility rates are expected to be above the value necessary for reproduction in the year 2050.

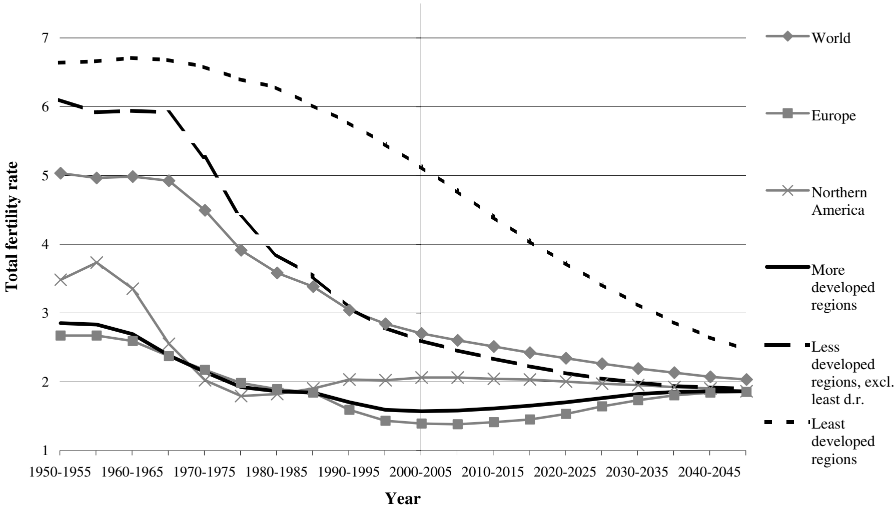
---

developed regions. Our classification corresponds to the UNDP's classification *less developed regions, excluding least developed regions*.

[4] See http://esa.un.org/unpp/Glossary.html.

[5] It is obvious why this value should be around two. The reason for a value above two is twofold. Firstly, the sex ratio at birth, i.e. the number of male births per one female birth, is slightly above unity and secondly, not all females who are born will reach childbearing age.

[6] Note that the notation 1950–1955 signifies the full period involved, from 1 July of 1950 to 1 July of 1955.



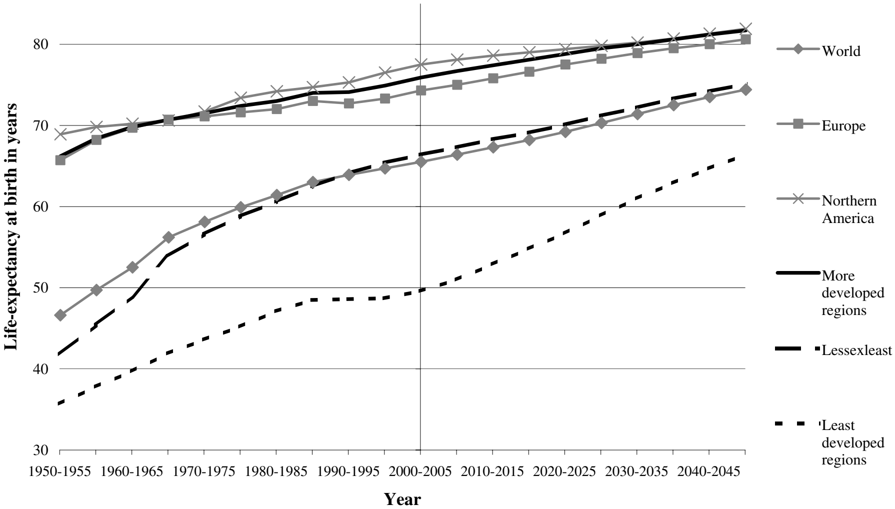
Source: United Nations Population Division (2003a), medium-fertility assumptions.

Fig. 3.1. Total fertility rates around the world: 1950–2050

Life-Expectancy. Trends in changes of mortality are best summarized by the life-expectancy at birth. Usually this indicator is differentiated by sex. For the sake of a clearer presentation we will only report the values for both sexes combined.[7] The definition of life-expectancy at birth is given by "the average number of years of life expected by a hypothetical cohort of individuals who would be subject during all their lives to the mortality rates of a given period. It is expressed as years."[8] In Fig. 3.2, the developments of the life-expectancy for different regions are presented. Again, the values for the years after 2000 correspond to the assumptions made in the medium variant projection of the UNDP. Under these normal-mortality assumptions, mortality declines at a medium pace in most developing countries. However, a slow pace of mortality decline is projected for countries that are highly affected by the HIV/AIDS epidemic.

The development of life-expectancy over time differs sharply between regions depending on the stage of economic development. Over the last five decades the *more developed regions*, notably Europe and North America, have experienced a rise of life-expectancy of roughly ten years; starting from values between 65 to 70 years. Under the normal-mortality assumptions a further increase of life-expectancy of six years to a level of nearly 82 years is expected for the *more developed regions*. While most developed countries are behaving quite similar in respect to the growth of life-expectancy, Japan has undergone the most severe changes: the life-expectancy has risen from 64 years in the period 1950–1955 to 82 years in the period 2000–2005 and

[7] In the *more developed regions* women currently have a ten percent higher life-expectancy at birth than men do. This differences tends to rise with the economic development of a country.
[8] See http://esa.un.org/unpp/Glossary.html.



Source: United Nations Population Division (2003a), medium variant.

Fig. 3.2. Life-expectancies at birth around the world: 1950–2050

it is expected to rise to 88 years in 2045–2050. Japan is the country with the highest life-expectancy today and its population is expected to remain the longest-living in the entire world for the next 50 years.

The decrease in mortality has been even larger in size for the *less developed regions* both in absolute and in relative terms. The life-expectancy at birth in these regions has risen from an average of 42 years in 1950–1955 to an average value of 66 years in the 2000–2005 period. That constitutes a rise of 25 years or nearly 60 percent. In comparison, life-expectancy in the *more developed regions* has only risen by ten years or 15 percent during the same time span. For the *least developed regions* mortality fell at nearly the same rate as in the *less developed countries* until the middle of the 1980s. Since then life-expectancy has unfortunately stagnated at a level of less than fifty years mainly due to the HIV/AIDS epidemic. Even though this trend is expected to be broken by 2010, with the incidence of HIV infection projected to decline after this date, life-expectancy in these regions is expected to be drastically lower than in the rest of the world in 2045–2050.[9]

3.1.2 Changes in the Population Size and the Age Structure

The combination of continuously sinking mortality and low fertility has a severe influence on the age structure of the population. In order to demonstrate the influence of the demographic transition on the age structure of the population and the population

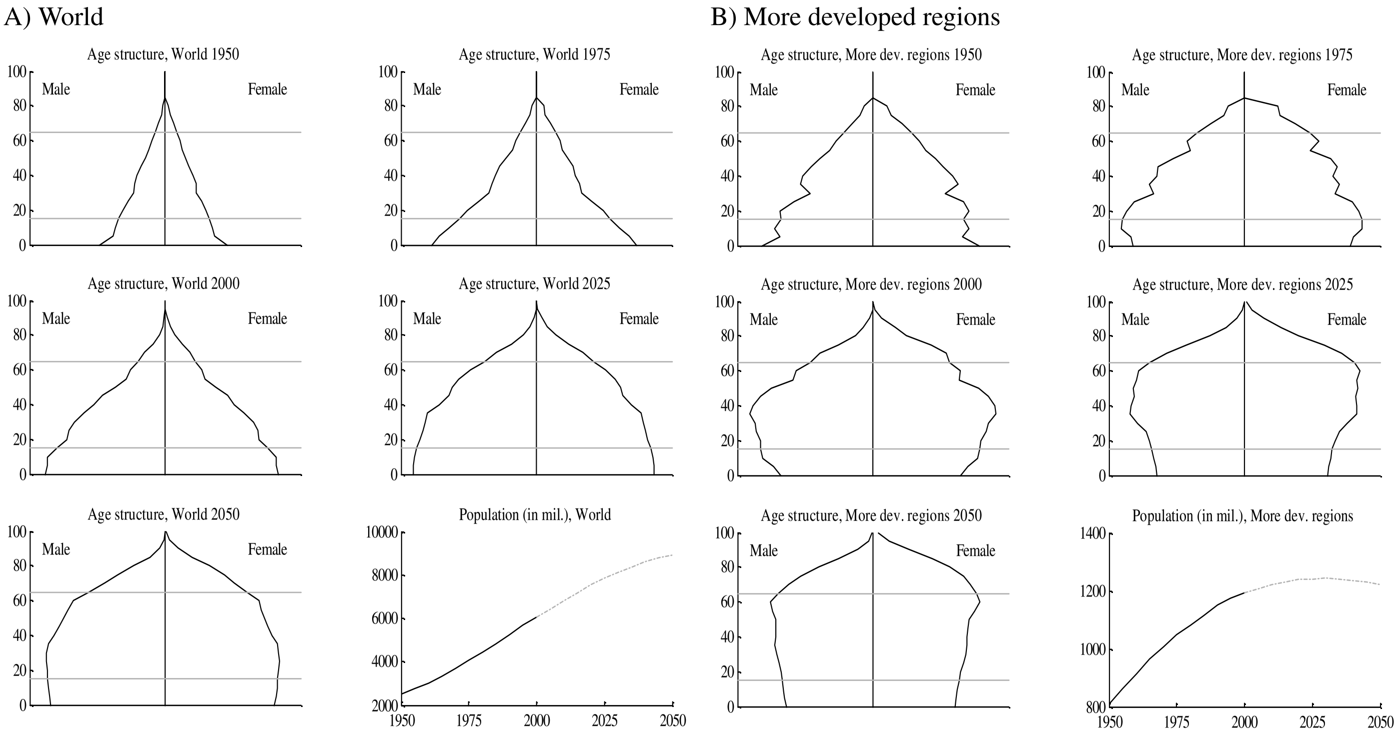
[9]Compare United Nations Population Division (2003b, pp. 10–14) for the demographic impact of HIV/AIDS.

size, we display the population pyramids and the size of the total population for different regions in Fig. 3.3. The figure shows the size of the male and female population by five-year age groups for the years 1950, 1975, 2000, 2025, and 2050 for different regions (A World; B More developed regions; C Less developed region, excluding least developed regions; D Least developed regions; E Europe; F North America; G OECD; H G7). The scaling is kept constant over time for each respective region. Furthermore, grid-lines are included at the age of 15 and 65 to illustrate the division of the population into youths, working-aged, and elderly. Also, the development of the total population for each region is portrayed in the sub-figure in the lower right corner of Figures 3.3A–3.3H.

In Fig. 3.3A, the entire world population is depicted. The top two sub-figures (1950 and 1975) show why these illustrations are called population "pyramids". The silhouette of the age structure has the shape of a triangle or pyramid, because generally, a cohort born in a given year tends be larger than the cohort born in the previous year. This is synonymous to fertility rates that are constantly above the reproduction level, i.e. positive long-run population growth rates. In the sub-figure for the age structure of the world in 1975, one can observe how the high fertility rates after World War II have led to a rapid population growth of the young population: the population pyramid becomes flatter for age-groups younger than 25. From the current age structure (year 2000) one can confirm firstly, that fertility has been substantially lower in the past ten to fifteen years than it had been before (a kink in the line of the population pyramid at the age-group of the 10 to 14 year old) and secondly, that mortality has declined (larger cohorts for age 65 and older). The predicted age structure in 2050 shows that in the future one will no longer be able to speak of a population "pyramid" but much more of a population "Taj Mahal".

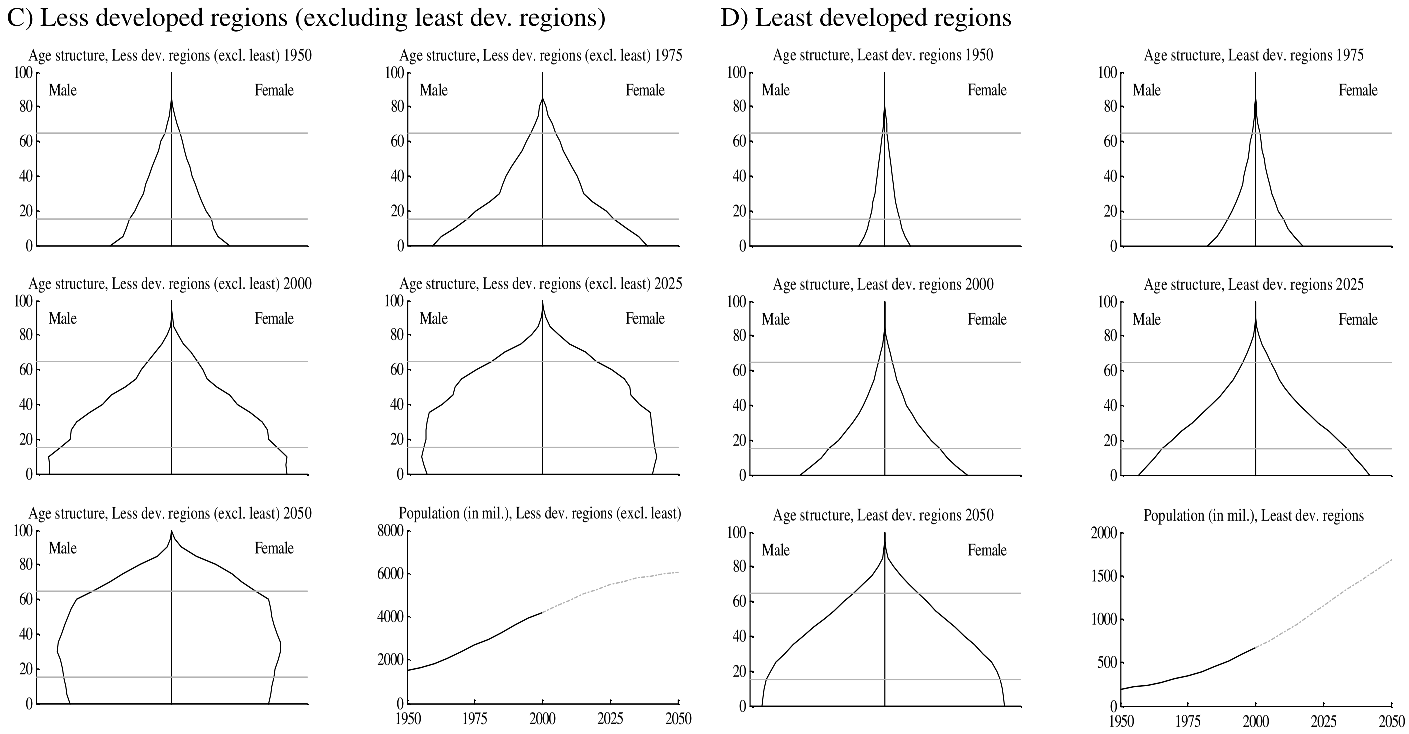
Comparing Figures 3.3A and 3.3B shows that the process of demographic transition in the *more developed regions* has a time-lead of about 50 years in comparison to that process in the entire world. Further comparison with Figures 3.3C and 3.3D indicates that the changing of the demographic composition of the population of the entire world is more or less given by the dynamics of the *less developed regions*. As mentioned, the *more developed regions* lead the aging process by roughly fifty years and the *least developed regions* lag that process by fifty years. Since the *less developed regions* represent roughly 70 percent of the world population (4.3 of 6.2 billion people in 2002), it is not surprising that these regions are dominating the dynamics of the demographic transition in the world.

As can be confirmed from Figures 3.3E through 3.3H, all sub-regions of the *more developed regions* are basically subject to the same development. Only Northern America differs slightly, since it experienced a larger baby-boom (high fertility rates) in the post-World War II decades, and the baby-bust period (below replacement fertility) was limited to a time-span of approximately 15 to 20 years between 1970 and 1990. Therefore, the age structure will be a little more favorable in Northern America than it will be in Europe and Japan (not depicted) over the next half century. The case of Europe (Fig. 3.3E) shows the looming demographic pressure between the years 2025 through 2050 most dramatically. Large cohorts of retirees (over 64 years) in



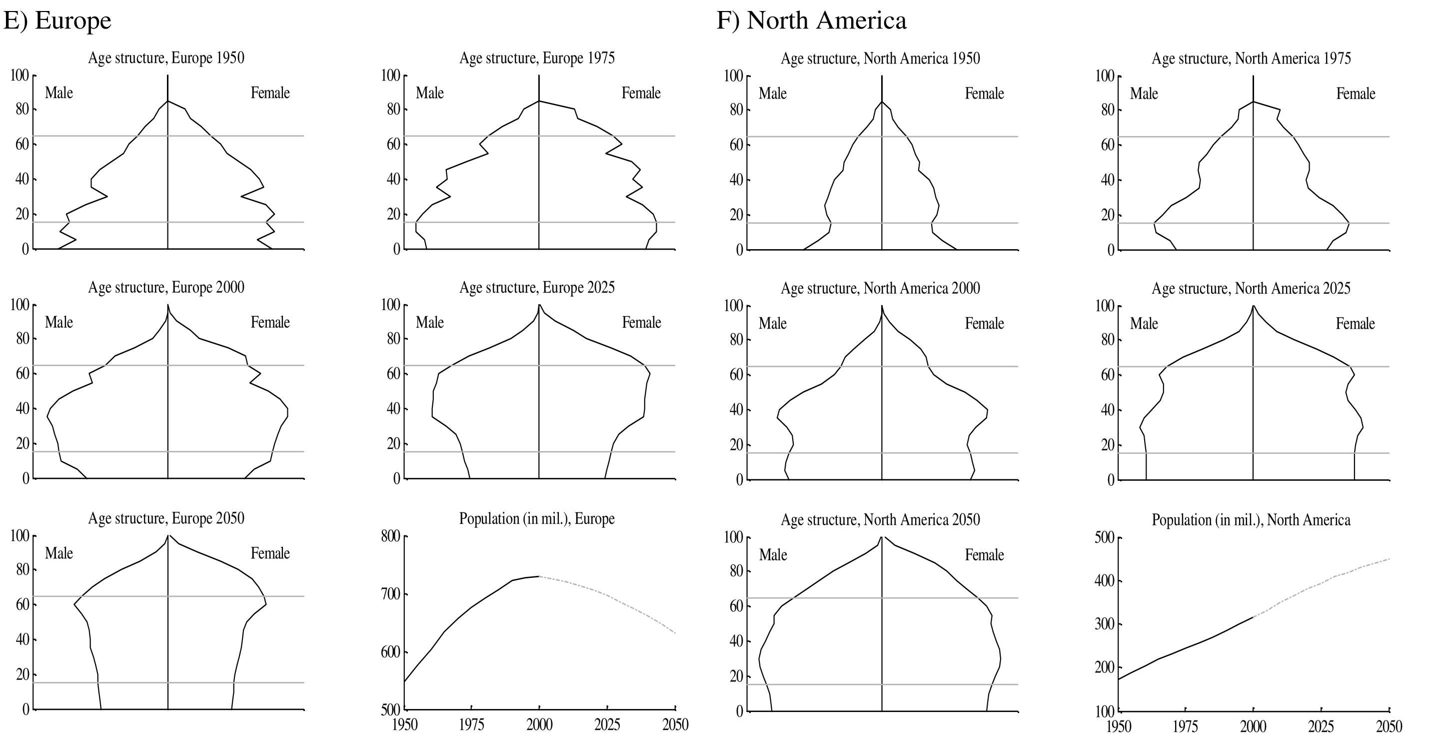
Source: Author's presentation using data from United Nations Population Division (2003a), medium variant.

Fig. 3.3. Age structure and population size over time for selected regions



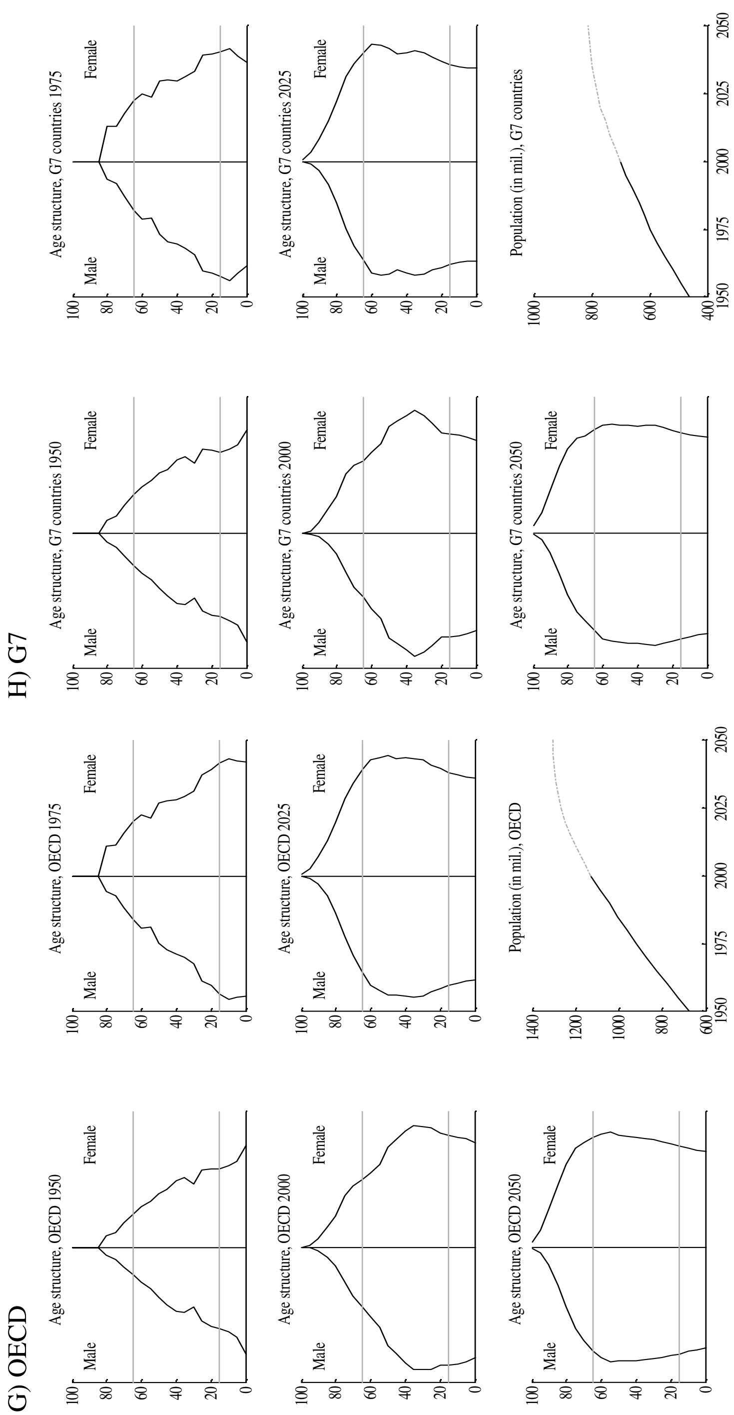
Source: Author's presentation using data from United Nations Population Division (2003a), medium variant.

Fig. 3.3. (Continued)



Source: Author's presentation using data from United Nations Population Division (2003a), medium variant.

Fig. 3.3. (Continued)



Source: Author's presentation using data from United Nations Population Division (2003a), medium variant.

Fig. 3.3. (Continued)

the top section of the population pyramid – or rather the population "mushroom" – go together with small and continuously decreasing cohort sizes of the working-aged population (15–64). Finally, we can confirm that especially the industrialized countries of the OECD (see Fig. 3.3G) and the most industrialized countries that make up the G7-group (see Fig. 3.3H) are currently undergoing the demographic transition towards an aging society.

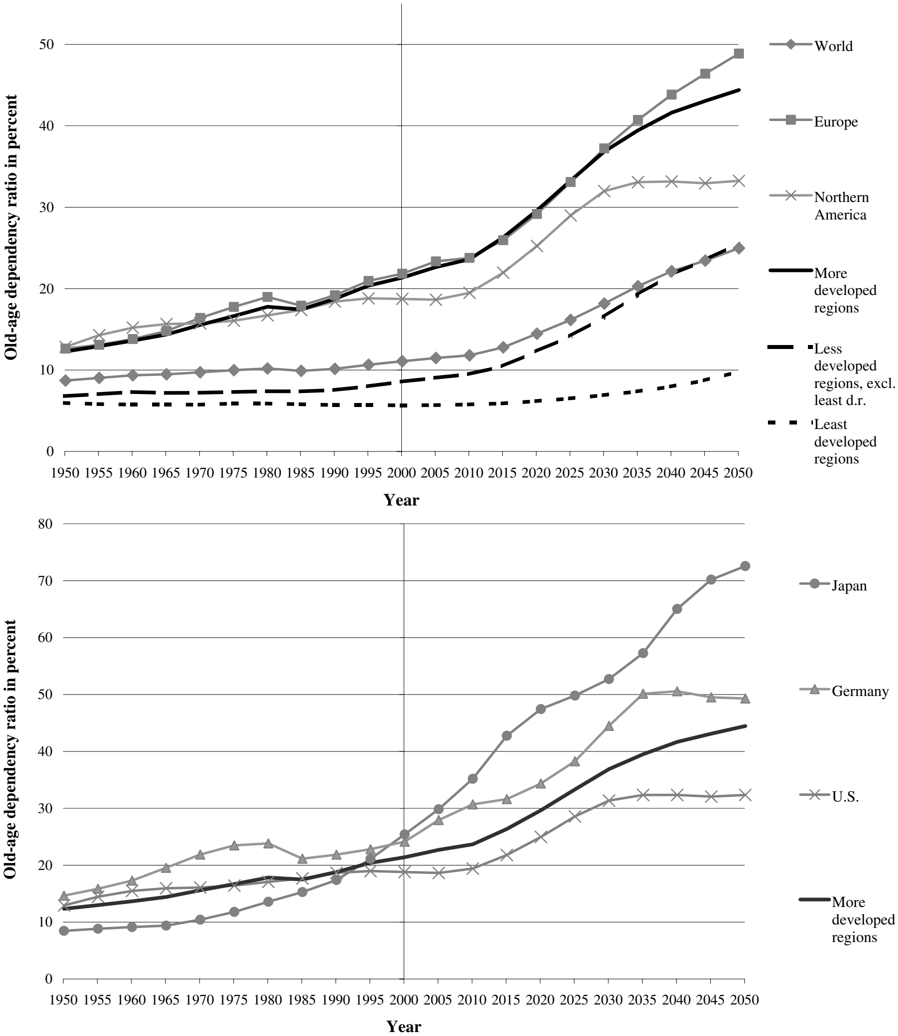
3.1.3 The Old-Age Dependency Ratio: 1950–2050

Population aging can be represented by indicators that summarize the age structure into one numeric value. Meaningful indicators are the median age and dependency ratios. For the purpose of discussing the impact of population aging and social security, it is helpful to concentrate on the old-age dependency ratio (OAD). This ratio measures the relative size of the elderly population (in our case the above 64 year old) as a proportion of the working-aged population (15 to 64 year old). This ratio is the most instructive indicator in the context of social security because it shows how many potential retirees have to be supported by the potential work-force.[10] In Fig. 3.4, the time path of the old-age dependency ratio according to the medium variant projection of the UNDP is presented. The top panel shows the different regions as classified in Sect. 3.1.1 and the bottom panel displays the old-age dependency ratio for the three largest economies in the world: the United States, Japan, and Germany. Note that the scaling of the vertical axis differs between the top and the bottom panel. To facilitate comparison, we have included the OAD's trajectory for the *more developed regions* in the lower panel as well.

The absolute values for the OAD in the year 2000 are 21.2 percent, 8.6 percent, and 5.7 percent for the *more*, *less*, and *least developed regions*, respectively. In the past 50 years the OAD has increased by 75 percent, 26 percent, and -5 percent in the respective regions. For the world, this ratio has increased from 8.6 percent in 1950 by 28 percent to a value of 11.0 percent today. The future old-age dependency ratio is dependent on today's age structure, future mortality and birth rates, and the pattern of international migration. The assumptions concerning the total fertility rate and life-expectancy were cited in Sect. 3.1.1. For the assumptions on international migration, see United Nations Population Division (2003b). Note that for a medium-term time horizon the projection of the OAD is not very sensitive to the specific demographic assumptions. Different assumptions on fertility will, by definition, only have an effect on the values forecasted 15 years into the future.

Over the next 50 years the OAD is expected to more than double in the *more developed regions* and nearly triple in the *less developed regions*. In 2050, roughly 44 (26) persons over the age of 64 must be supported by 100 persons that are in the age of being potentially a part of the work-force in the *more (less) developed regions*. Taking

[10] For the *more developed regions*, it is common to express the old-age dependency ratio as the ratio of above 59 year old as a fraction of the 20 to 59 year old. This is done because longer education duration delays the entry into the work-force and early retirement options allow an earlier exit of the work-force.



Source: United Nations Population Division (2003a), medium variant.

Fig. 3.4. Developments of the old-age dependency ratios around the world

account of the female participation rate and further reduction of the work-force due to education, unemployment, and early retirement, a scenario of one retiree per worker seems quite realistic for the *more developed regions*.

In the bottom panel of Fig. 3.4, the old-age dependency ratios of the three largest economies – Germany, Japan, and the United States – are depicted. Again, we can confirm that Japan is facing the most severe aging of its population. While the values for the OAD is currently in roughly the same region for the three countries, it is expected that Japan will already be faced by values of over 50 percent by the year 2025 and will likely reach values over 70 percent. It is notable that Germany and Japan have undergone quite similar trends in fertility. However, Germany will probably only reach an old-age dependency ratio of around 50 percent for the years 2030 and beyond. The difference is due to substantial higher net immigration in Germany[11] and a far higher life-expectancy in Japan. The projection of the old-age dependency ratio for the United States is a lot more favorable than those for Japan and Germany. This is a result of higher fertility in the past decades and higher expected net migration rates in the future.

3.2 Implications of Demographic Transition

The challenges for aging societies and the economic consequences of population aging have been widely discussed.[12] Not surprisingly, many of these studies focus on pension systems in an aging society. Here we concentrate on two topics. First, the impacts of aging on the sustainability of the public sector (Sect. 3.2.1). Second, macroeconomic effects on factor prices should be expected during demographic transition (Sect. 3.2.2). In a welfare analysis of pension reforms, factor price effects should be taken into account. To what extent international capital flows can alleviate strong factor price movements is also shortly addressed. Finally, it has been asserted that the aging of the Baby-Boomers may affect asset prices due to the changing demand for assets over the life-cycle of the Baby-Boomers. We summarize some findings on this so-called “asset market meltdown hypotheses”.

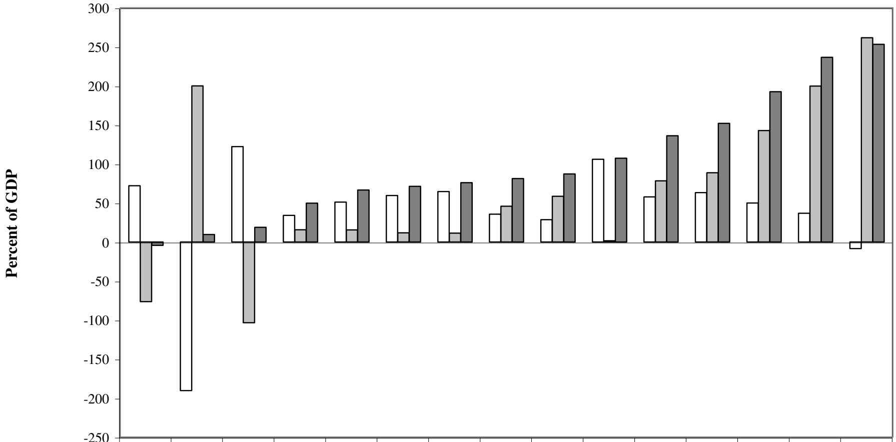
3.2.1 Aging and Fiscal Sustainability

The drastic increase of the old-age dependency ratio has far-reaching implications for the future sustainability of government tax and transfer programs. Public pension programs that rely on PAYG financing are the most prominent example for a “demographic squeeze” on fiscal coffers. However, other programs such as health insurance will be affected in a similar fashion: health care expenditures exhibit an age-profile that sharply increases with age.[13] In addition, some countries, also make health insurance premiums dependent on wage income. As a result health care expenditures will

[11] According to the medium variant projection of the UNDP, the net migration rate per 1000 inhabitants is expected to be 2.6 in Germany and around 0.5 in Japan. In absolute terms, the net annual migration equals 211,000 and 54,000 for Germany and Japan, respectively.

[12] See e.g. Bös and Weizsäcker (1989), Cutler et al. (1990) World Bank (1994), Disney (1996), Saas and Triest (1997), OECD (2000), and Holzmann and Stiglitz (2001).

[13] It is an open dispute whether health care costs are mainly dependent on absolute age or rather on the remaining time until death; cf. Zweifel et al. (1999). If the latter case is true, advances in longevity should not increase individual health care expenditures too much. However, Borgmann and Raffelhüschen (2004) show in a Swiss case study that for the matter of sustainability, this distinction is not very important. The simple fact that in the coming decades, an increasing fraction of the population will be approaching the last years prior to death will put severe strains on the health care system in either case.



	IRL	N	B	CH	GB	DK	NL	F	US	I	D	E	A	S	SF
explicit debt	72	-191	122	34	51	60	65	36	29	106	58	63	50	37	-8
implicit debt	-76	200	-103	16	16	12	11	46	59	1	78	89	143	200	262
true debt	-4	10	19	50	67	71	76	81	87	107	136	152	193	237	253

Source: Raffelhüschen and Borgmann (2001) and references reported there.
Notes: Growth rate 1.5 percent, discount rate 5 percent, indexation of transfers according to legal status quo.

Fig. 3.5. Results from generational accounting for the baseyear 1995: cross-country survey

increase and the receipts from insurance premia will decrease during the demographic transition.[14] Similar arguments can be made for unemployment insurance and for the receipts from income-taxes.

In order to quantify the sustainability of fiscal policy and welfare programs, Auerbach et al. (1991,1992,1994) have developed the method of generational accounting.[15] The method is an intertemporal accounting framework that uses the age-specific distribution of government receipts and expenditures together with demographic projections to quantify a country's "true debt" given that the current legal status quo is kept in place ad infinitum.[16] The "true debt" – or alternatively "sustainability gap" – is composed of the explicit debt and an implicit debt. The implicit debt is the part that captures the burdens from e.g. the perpetual continuation of PAYG social security.

In Fig. 3.5, taken from Raffelhüschen and Borgmann (2001), the results from cross-country studies are summarized (cf. Jägers and Raffelhüschen (1999), Raf-

[14] In a related matter, Feldstein and Samwick (1997) find that pre-funding Medicare benefits in the U.S. would be of analogous magnitude as pre-funding social security.

[15] Generational accounting also provides information on the intergenerational distribution of burdens of reform proposals.

[16] For a description of the currently used indicators of sustainability and a critical discussion of the method, see Raffelhüschen (1999b) and Bonin (2001a). Besendorfer (2003, Chapter 1) surveys alternative approaches to measure fiscal sustainability. See also Banca d'Italia (2000).

felhüschen (1999a), Kotlikoff and Raffelhüschen (1999), and Gokhale and Raffelhüschen (1999)). The results show that for a number of developed countries the implicit debt is at least as large as the explicit debt. Moreover, in four of the 15 countries the implicit debt exceeds the size of annual GDP. The results are sensitive to the fiscal performance of the base year and the choice of parameter values, i.e. growth rate and discount rate. The discount rate equals five percent. For smaller values the implicit debt – measured in percent of GDP – will tend to be larger. Furthermore, we have applied the legal status quo for the indexation of pension benefits. In the countries where the indexation of pensions is less than wage indexation – Switzerland, United Kingdom, and France – future discrete jumps in benefits should be expected. Otherwise, benefits from the public pension program may fall short of the subsistence-level. Thus, the implicit debts for the respective countries tend to be underestimated here; cf. Raffelhüschen and Borgmann (2001) and Raffelhüschen (2002).

The presented results are for the entire public sector including welfare programs. For the remaining part of this book we will concentrate on old-age provision. The reader should bear in mind that especially health care and long-term health care programs are faced by similar demographic pressures. In Sect. 6.5, we again resort to the method of generational accounting to study the sustainability of isolated social security for Germany.

3.2.2 Aging, Factor Prices, Capital Flows, and Pension Reform: A Survey

From a macroeconomic perspective one can expect that the process of population aging will have an influence on the productive capacities of the economy. Labor will become increasingly scarce while capital will be relative abundant. Hence, the capital-labor ratio will most likely increase. In a competitive economy, this will affect factor prices during the demographic transition: a rise in the wage rate and a decrease in the interest rate should be expected. Saving behavior and labor supply decisions of private households may change in anticipation of these future developments. In addition, current pension reforms and the prospect of future pension reform may influence national saving and individual behavior today.

The framework to study the effects of demographic transition on the saving behavior, the capital stock, and hence factor prices has been put forward by Auerbach and Kotlikoff (1987) in their book *Dynamic Fiscal Policy*.[17] Using computational methods, it has become feasible to study sophisticated models of overlapping generation in dynamic general equilibrium. Recent examples of computable dynamic general equilibrium models with overlapping generations, where pension reforms during demographic transition are analyzed, can be found in Kotlikoff et al. (1999) for the United States, Miles (1999) for Great Britain and Europe, and Börsch-Supan et al. (2002a) for Germany. Extended versions of the Auerbach-Kotlikoff model also allow the inclusion of intragenerational heterogeneity. Fehr (2000) uses such a model to quantify the welfare effects of pension reform proposals in Germany.

[17] For an early application of a computable, dynamic, general equilibrium model addressing questions of social security, see also Raffelhüschen (1989b).

Pension Reform During Demographic Transition and Welfare. The basic question that has been addressed in a variety of differently specified general equilibrium models with overlapping generations is the following: should the social security system be (at least partially) funded in order to tackle the challenge of an aging economy?[18] A simple but insightful answer to this question can be found in Raffelhüschen and Risa (1997).[19] The authors come to the conclusion that funding social security during a baby-bust scenario is either welfare decreasing or time inconsistent. In this context time inconsistent means that if the social discount rate of a utilitarian planner is low enough for funding to be a welfare increasing strategy during demographic transition, than funding should have already been welfare-improving under stationary demography. The general intuition is the following: while small generations, i.e. Baby-Busters, are most likely faced by a very low return on their social security contribution, it is nevertheless the generation of the Baby-Boomers who experience the lowest life-cycle utility due to the macroeconomic effects on factor incomes during demographic transition. This is especially the case in a baby-boom baby-bust scenario, where the Baby-Boomers will be subject to a low wage rate when young[20] and a low interest rate when old. At the same time, the Baby-Busters profit from a relative high wage rate during their working period. Undertaking pension reform during demographic transition that lead to more capital accumulation, such as pre-funding social security or switching to a defined contribution scheme, will additionally increase the factor-income changes. Thus, Baby-Boomers may bear a double-burden from pension reform: first, many pension reform proposals steer towards a reduction in the generosity of the pension benefits paid to the Baby-Boomers and second, the pension reform tends to amplify the adverse factor-income effects.

However, Bütler and Harms (2001) show that the specification of the model can have a severe impact on the magnitude of the predicted swings in factor prices. The number of generations, endogenous labor supply, and convex adjustment costs attenuate factor price effects. Using a model with 80 overlapping generations for Germany, Börsch-Supan et al. (2002a) come to the conclusion that the decrease of the rate of return from pension reform during demographic transition is often overstated. Nevertheless, Bütler and Harms (2001), Bohn (2001), and Young (2001) all come to the conclusion that because of general equilibrium effects on factor returns, large generations are most likely to be hit hardest by the demographic transition. Bohn (2001) concludes from this result that reforming social security such that the Baby-Boomers are made even worse off, does not seem to be a welfare-improving policy.

Political Economy of Social Security. The influence of demographic transition will most likely also have an impact on the political process of pension reform itself. The

[18] A closely related matter is the question of whether the social security scheme should be switched from defined benefits to defined contributions.

[19] Even though the authors are mainly addressing the issue of whether generational accounting is a good instrument to analyze fiscal policy in a closed economy, the question they answer is exactly the one asked above.

[20] Welch (1979) provides empirical evidence that wage rates have indeed been depressed for the baby-boom cohorts.

political economy of social security studies the electoral results on pension policies depending on the age structure of voters.[21] Naturally, the age of the median voter will rise during demographic transition. This may lead to changing majorities in the political process with respect to social security legislation. At the same time, the ever increasing burden of pay-roll contributions for the PAYG social security scheme may lead to a situation where the working population is opting out of social security in one way or the other. To make things even more complicated, the elderly part of the population may anticipate that some threshold-value of maintainable social security contributions exists and use their political power only to a degree that will keep contributions underneath the threshold level; cf. McHale (2001). Obviously, the political process of determining the size of the public pension program will be severely influenced by the demographic transition. A recent survey of the literature addressing the political economy of social security can be found in Galasso and Profeta (2002).

International Capital Flows. As has been illustrated in the first Section of this Chapter, population aging is a world-wide phenomena. However, not all regions in the world are affected equally. Size and timing of this phenomena differs substantially between countries and regions. From Fig. 3.4, one can see that the world's *more developed regions* are roughly 40 to 50 years ahead in the process of aging in comparison to the *less developed regions* of the world. And even within industrialized countries, a considerable time-shift can be observed. With integrated financial markets, international capital flows might partially alleviate the macroeconomic reactions on factor incomes described above.[22] In recognition of the significance of the macroeconomic impact of population aging on a global level,[23] various research groups have engineered multi-region general equilibrium macroeconomic models: the OECD has developed the MINILINK (see Turner et al. (1998)), a team of french researchers has developed the INGENUE model (see INGENUE (2000)), and Börsch-Supan et al. (2002b) have put forward a multi-region overlapping-generations model as well. A more stylized model which only covers four overlapping generations can be found in Brooks (2000a). To be precise, the MINILINK is not a life-cycle model. Life-cycle effects are, however, partially captured in the spirit of Blanchard's (1985) model of "perpetual youth". Börsch-Supan et al. (2002b) mainly focus on capital flows within the EU countries and OECD countries. However, in one experiment they show that the time-path of the interest rate is nearly identical for the OECD-case and the world-scenario.

A common result of these studies is the prediction of capital flows between the world regions in the coming decades: Europe and Japan will experience current account surpluses until roughly 2020 to 2030. Beyond this time, the current account switches to deficits in these regions. The trend of the U.S. current account is similar to the European and Japanese experience, but less in magnitude. Also, current account deficits will not prevail for very long after 2030 since aging is much less profound

[21] Browning (1975) showed in his seminal work that social security tends to be large in democracies.

[22] The same holds for asset prices which are discussed below; see also Goyal (2002).

[23] Higgins (1998) uses time series econometrics to show that national savings and current account balances are in fact dependent on the demographic structure of the population.

in the United States. The international capital flows are in large parts the result of different national savings behavior due to the population aging. Net foreign wealth is accordingly build up during the years where Baby-Boomers are in the prime-saving ages (40 to 65) and the net foreign assets will be decumulated when the large cohorts are retired. Brooks (2000a) shows that the general direction of the results of predicted capital flows are fairly robust regarding different assumptions of convergence within the different regions in the world economy.

The large capital flows help to smooth the accumulation cycle arising from population aging. The factor price swings, especially that of the interest rate, will therefore be much less drastic than predicted in closed economy models. The results of Börsch-Supan et al. (2002b) show that under the current pension system the interest rate reduction is projected to be about 0.5 percentage point lower in 2030 if Germany can invest internationally in comparison to a closed economy case. Fundamental pension reform will decrease the interest rate by another 0.5 percentage points without international capital mobility. In the scenario with perfect international capital mobility within the OECD countries, the German pension reform has hardly any effect on the "world" interest rate. Thus, the reduction of the interest rate will be about one percentage point larger for the case of pension reform in a closed economy in comparison to a scenario of a pension reform with international capital flows. Similar results are derived by INGENUE (2000, Chart 14). In the pension reform scenario where contributions are held constant throughout Europe (comparable to partial funding), the interest rate would decrease by an additional percentage point if Europe is assumed to be a closed economy in comparison to the benchmark of perfectly integrated world financial markets. INGENUE (2000) compare their results from the world model against the assumptions of a closed economy and a small open economy. Their results indicate that the former approach completely ignores the effects of global financial integration on domestic developments, whereas the latter framework over-emphasizes these effects. An analysis of pension reform during demographic transition in a closed economy thus strongly overestimates factor-price effects. Hence, the critical standpoint towards pension reforms during demographic transition cited above have to be taken with a grain of salt.

The above presented results on international capital flows are subject to several limitations. First, capital markets are assumed to be perfectly integrated, such that domestic and foreign capital are perfect substitutes. This contradicts the well known empirical results from Feldstein and Horioka (1980), Frankel (1991), and Taylor (1996), who find a strong correlation between national saving rates and national investment rates. The "home bias" in equity ownership that can be observed despite a large volume of cross-border capital movements, see e.g. French and Poterba (1991), confirms this notion. Thus, the predicted international capital flows have to be interpreted as an upper-bound. Second, as always, some restrictions apply to the ultra-rational behavior of agents within the approach of computable general equilibrium models and to the life-cycle behavior according to the life-cycle hypothesis. Third, most of the above studies ignore aspects of financial-market risk and exchange rate risks; cf. Ulrich Grosch's discussion of Börsch-Supan et al. (2002b) in Auerbach and Herrmann (2002, pp. 90–95).

Finally, the conclusions on the benefits of international capital flows for coping with population aging differ quite substantially between the different studies. Börsch-Supan et al. (2002b, p.58) are very optimistic that "Capital exports to developing countries could therefore solve the aging problems of industrialized countries by reducing pressure on the interest rate." In contrast, Turner et al. (1998, p.29) conclude that "increased foreign investment is not a general panacea and can make only modest contribution to offsetting the projected decline in the growth rate of living standards relative to past trends." According to these authors, only a basket of measures and reforms, finely tuned in timing and international coordination, can help to limit the unfavorable macroeconomic consequences of population aging.

Holzmann (2000) identifies three main advantages of investments in emerging markets: risk diversification, rate of return increase, and an improved economic environment.[24] The first of these three will be dealt with in Sect. 7.1.3. The second advantage is the argument captured by the dynamic overlapping generations models described above. The last effect builds on the assertion that stronger capital flows will enhance corporate governance and foster financial market developments, especially in emerging markets. This "financial deepening" will in turn contribute to higher growth-rates in these regions which is assumed to be beneficial for the developed world as well. Generally, Holzmann (2000) is also skeptical towards the potentials of investing in emerging countries to solve the aging problem: he concludes that investments in emerging markets can help at the margin to attenuate the problem, however, these investments are unable to solve the problem. In addition, various politically and economically challenging requirements must be met in order to reap even the marginal gains. Specifically, the aging countries have to foster an increase of savings, e.g. by moving from unfunded to funded pension schemes. For the emerging markets, the danger lies in using enhanced capital flows to increase consumption instead of accomplishing increased investments into the capital stock. Reasonably developed financial markets in emerging markets are also a necessary condition in the first place.

Demographics and Asset Prices: The "Asset Market Meltdown Hypothesis". Next to a lower rate of return due to a rising capital-labor ratio in the production process, the return on capital investments may be further reduced by changes to the valuation of assets. The hypothesis is that changes to the supply and demand of assets due to demographic transition may have a severe influence on asset prices.[25] Accordingly, the implications of aging for the stock market are twofold: first, it has been asserted that the stock market boom of the late nineties was driven by the growing demand for financial assets from the Baby-Boomers' savings for retirement. Second, as the future counterpart of the phenomena, fears of the so-called "asset market meltdown" have been uttered. The prediction is that the baby-boom cohorts will all simultaneously try to cash out of their equity savings. Hence, due to the surge

[24] See also Reisen (2000) on capital flows from aging to emerging markets.
[25] See Yoo (1994) and Bergantino (1998) for early studies on this issue.

in the supply of stocks unmatched by an equivalent increase in demand, asset prices will plunge. Risky investments for individual old-age provision and pre-funded parts of social security might therefore loose in value when they are actually needed.

The argument was initially focused on the housing market.[26] In an early empirical study for the U.S. housing market, Mankiw and Weil (1989) find that a one percent increase in housing demand will lead to a five percent increase in real prices of houses. For the return of financial assets, Poterba (2001b) finds little evidence of a robust empirical relationship linking demographic structure to asset returns.[27] Poterba also provides some arguments why the asset meltdown should not occur. Firstly, the Baby-Boomers are not going to sell all their assets at one point in time. Instead, the liquidation of assets is a continuous process stretched over more than a decade. Secondly, the "news" about demographic changes is revealed with the birth of the cohorts. Rational, forward looking financial markets should thus not be caught by surprise by the Baby-Boomer's aging process. Asset prices should respond after the revelation of the news and not decades later. Overall, Poterba rejects the asset meltdown hypothesis on the basis of his empirical evidence. Campbell (2001, p.588) in his discussion of Poterba's paper concedes that "James Poterba is correct to conclude that demographic effects on future risky asset demands are likely to be modest, and hard to disentangle from the many other forces that influence asset markets." In contrast, Geanakoplos et al. (2002) point out that turning points of stock prices and price-earning ratios move parallel to the demographic cycle measured in medium-to-young ratio for the U.S. and Japan. Speaking in favor of the long-run predictability of stock-markets, Bergantino (1998) also presents evidence linking the level of real stock price to demographic changes.

Results from the literature of asset prices in dynamic general equilibrium models point in the direction that population structure should have some effects on asset prices and returns. Simulating a stylized general equilibrium model, Brooks (2000b) shows that asset-returns depend on demographic shocks. However, the effects are modest in magnitude for plausible-sized changes of population growth. Abel (2003) shows analytically in an OLG model that the baby-boom causes an increase in the price of capital and that the price of capital is mean-reverting.[28] Interestingly, social security will not effect the price of capital in the long run. On the other hand, Lim and Weil (2003) find in a forward-looking macro-demographic model that the meltdown should only occur when installation costs for capital are sufficiently large. These authors infer that conventional measures of installment costs are too small to explain large stock price movements due to demographic developments.

[26]Housing markets are likely to lead the cycle of asset price effects of aging because demand for housing rises sharply between the ages of 20 to 30 and increases modestly until 40. In contrast, accumulation of financial wealth is mainly focused in the age of 40 to 64.

[27]However, a synthetic "predicted asset demand" constructed from population variables yields "some evidence that higher asset demand can be associated with higher asset prices, as measured by the price-to-dividend ratio for common stocks" Poterba (2001b, p.566).

[28]In another paper Abel (2001b) shows that the inclusion of bequests does not overturn the results.

To conclude, the dispute on the influence of demographic transition on asset returns and asset prices remains unsettled. This is the case from an empirical point of view and, although to a lesser extent, from a theoretical point of view. While some effects can be expected, the magnitude of these effects should not be over-estimated.

Chapter 4
Old-Age Provision and Uncertainty: An Introduction to Issues of Risk and Risk Sharing

In this chapter we show why the introduction of uncertainty is important for understanding the economics of old-age provision. First, we construct a very simple model with one stochastic element, in which private markets fail to spread risk between generations. We thus show that social security may improve efficiency by providing *intergenerational risk sharing*. Second, a classification of the different risk components of old-age provision is presented.

This chapter mainly serves to set the stage for the remaining parts of this book. The objective is to provide the intuition how intergenerational risk sharing can help to improve welfare and that different aspects of risk have to be considered. We conclude by giving a guideline for Chaps. 5 through 7 where various specific aspects of risk in old-age provision are addressed in detail.

4.1 A Primer on Intergenerational Risk Sharing and Social Security

Until now, we have only considered economies without uncertainty about future outcomes. In particular, the development of labor income, asset returns, and demography were all deterministic. Without uncertainty about future developments, there is no need to insure oneself against possible bad realizations of future states of the economy. However, in a world with uncertainty, this is exactly what PAYG social security (and public debt) can provide: insurance.

The argument can be outlined as followed: consider an overlapping generations model with a stochastic process for the long run development of labor-productivity. The length of a cycle is roughly equal to 30 years. The young will work during their young period, but uncertainty about their labor income is only revealed at the end of the period. When young, a generation is exposed to the risk that an idiosyncratic shock on labor productivity will generate a high or low wage rate. In general this generation wants to insure itself against a possible bad outcome. However, there is no other generation alive to make such a contract with: the current old will be dead in the next period and the next generation is not born yet. When the next generation is born in $t+1$, the uncertainty of labor income for the generation born in t has already been revealed. After the realization of an uncertain outcome, an insurance contract will make no sense since at that point in time such a contract is not in the interest of one of the two parties. One can see that the argument is very similar to the missing markets argument of Diamond (1965): without government intervention, contracts that improve welfare are not feasible, because no two generations are alive both at the time when the contract would have to be signed, and at the time when the contract would have to be fulfilled. However, in contrast to Diamond's case, the inefficiency here is not due to non-optimal over-saving but due to non-optimal under-insurance. Only the government can implement an institution that enables the generations to share risk among each other.

This argument of intergenerational risk sharing was first made by Weiss (1979), focusing on the effects of money supply. An excellent exposition in a more general context can be found in Gordon and Varian (1988). Fisher (1983), Gale (1990), and more recently Barbie et al. (2001) analyze risk sharing implications of public debt, while Enders and Lapan (1982), Merton (1983), Hansson and Stuart (1989) consider intergenerational risk sharing in social security programs. Thøgersen (1998), and Wagener (2003a,b) very specifically look at the intergenerational risk sharing features of different policies for PAYG social security. Thøgersen argues that only wage-indexed pension schemes can provide intergenerational risk sharing. Wagener (2003b) shows that this result is sensitive to the underlying welfare concept. In a similar context, however, with a stronger focus on risk diversification instead of intergenerational risk sharing, Hauenschild (1999) and Matsen and Thøgersen (2004) take a portfolio approach to derive the optimal design of social security. Richter (1993) argues that under the existence of an asset that is supplied in fixed quantities and yields a positive dividend at all points in time, i.e. land, social security is not necessary to provide optimal risk sharing between generations. More generally, Demange (2002) shows that the introduction of social security is never Pareto-optimal when land exists and financial markets are sequentially complete. Since sequentially complete markets constitute a rather unrealistic assumption, it is nevertheless worthwhile to study the risk sharing of social security, see e.g. Krueger and Kubler (2002). Bohn (1997, 1998) compares public debt and social security as instruments to share risk between generations. In contrast to the result derived for the deterministic case, where public debt and social security are equivalent under specific conditions, Bohn comes to the conclusion that under uncertainty there is a substantial difference between public debt and social security. In particular, he finds that a market solution allocates too much productivity risk on the young and too little on the old. Moreover, public debt shifts even more risk on the young, whereas social security is basically risk neutral.

To illustrate intergenerational risk sharing via social security, we will follow Thøgersen (1998) and augment the OLG model of Chap. 2 with a stochastic component for labor income. Assume an economy where the labor income of the young generation is composed of $w + \hat{\epsilon}_t$, where $\hat{\epsilon}_t$ is an independent identically distributed (i.i.d.) shock to productivity with an expectation of 0 and σ^2 variance. To keep things simple, we further assume that wage and interest rates are independent of the capital stock and that interest rate and population growth are both zero.[1] The period budget

[1] This case is a special case of a golden rule steady state since $r = n$. For social security issues this specific golden rule steady state is equivalent to all other golden rule steady states.

constraints (depicted P1 and P2 below) for the fully funded system and for PAYG social security are very similar to equations (2.5) through (2.7) with $r, n = 0$ and a stochastic term $\hat{\epsilon}_t$. Note that before the social security payment was a lump-sum payment τ. Here, the contribution $\tau = \gamma[w + \hat{\epsilon}]$ is a fixed proportion of labor income.[2] Making the social security benefit a fixed proportion of the young generation's labor income will be the instrument that helps to share risk amongst generations.[3]

The benefit payment in the old period will then be the same fixed proportion γ of the next generation's labor income when young. Benefit payments are given by the social security budget constraint: $\beta_{t+1} = \gamma(w + \hat{\epsilon}_{t+1})$.[4] The periods' budget constraints together with the social security budget constraint then yield the life-cycle budget-constraints (LC, last line below) under the two systems:

	funded (f)	pay-as-you-go (ss)
P1	$c_t^{y,f} = w + \hat{\epsilon}_t - s_t^{y,f}$	$c_t^{y,ss} = (w + \hat{\epsilon}_t)(1 - \gamma) - s_t^{y,ss}$
P2	$c_{t+1}^{o,f} = s_t^{y,f}$	$c_{t+1}^{o,ss} = s_t^{y,ss} + \beta_{t+1}$
LC	$c_t^{y,f} + c^{o,f} = w + \hat{\epsilon}_t$	$c_t^{y,ss} + c_{t+1}^{o,ss} = w + (1 - \gamma)\hat{\epsilon}_t + \gamma\hat{\epsilon}_{t+1}$

It is obvious that the expected value of life-cycle income before realization of ϵ_t is equal to w under both regime since $\mathrm{E}[\epsilon_t] = 0, \forall t$. However, the variance of life-cycle income differs. Under the fully funded scheme the variance of life-cycle income is equal to the variance of the productivity shock σ^2, while under the social security scheme the variance of life-cycle income equals

$$\mathrm{Var}[y_t^{LC,ss}] = (1 - \gamma)^2 \mathrm{Var}[\hat{\epsilon}_t] + \gamma^2 \mathrm{Var}[\hat{\epsilon}_{t+1}] + 2\gamma(1 - \gamma)\mathrm{Cov}[\hat{\epsilon}_t, \hat{\epsilon}_{t+1}]. \quad (4.1)$$

By assumption, $\hat{\epsilon}$ is i.i.d.. Thus, the covariance of shocks between periods is zero and the variances of the shock is equal to σ^2 in every period. The variance under social security therefore equals $[(1 - \gamma)^2 + \gamma^2]\sigma^2$. The value $\gamma = \frac{1}{2}$ minimizes this variance at $\frac{1}{2}\sigma^2$. The variance is reduced because the young share the outcome of their productivity shock with the current old and will in return participate in next generation's productivity shock $\hat{\epsilon}_{t+1}$. Because future generations are naturally excluded from financial markets, it is obvious why the market allocation could not pool the exposure to productivity risk over time. Only a government institution can do this.

Social security will reduce the variance of life-cycle income and will therefore increase welfare for risk-averse individuals. Of course, the unambiguous sign of the positive welfare change is somewhat constructed. As discussed in Chap. 2, a social security scheme imposes an implicit tax on the individuals. Hence, in a more realistic setting one should expect a reduction in expected life-cycle resources due to social security. Reintroducing non-zero interest rate and growth rate, the pension payments to the old are given by $\beta_{t+1} = \frac{1+n}{1+r}(w + \hat{\epsilon}_{t+1})\gamma$. Life-cycle resources will than equal

[2] All earlier results remain unchanged in a setting with social security contributions proportional to labor income. To verify this, simply substitute $\tau = \gamma w$ in the equations of Chap. 2.

[3] Thøgersen (1998) refers to this as the *fixed tax rate*.

[4] Note that $N_{t+1}^o = N_{t+1}^y$ from the assumption of zero population growth.

$y_t^{LC,ss} = w - \gamma w \frac{r-n}{1+r} + (1-\gamma)\hat{\epsilon}_t + \gamma \frac{1+n}{1+r}\hat{\epsilon}_{t+1}$. For an economy on a dynamically efficient growth path, expected life-cycle resources are less under a social security scheme than under a fully funded system: $\mathrm{E}[y_t^{LC,ss}] = w - \frac{r-n}{1+r}\gamma w < w$. An analysis of welfare effects of social security will therefore need to account for the trade-off between the welfare loss from the decrease in life-cycle resources and the welfare gain from the decrease in life-cycle resource variance.

4.2 Old-Age Income and Sources of Aggregate Risk

In the preceding "primer" to intergenerational risk sharing, we have only considered one very specific source of risk, namely labor income risk. In this section, we outline the different risk components of old-age income. We assume that fully functional insurance markets exist for risks concerning individual longevity, disability, unemployment, and health care. As the literature on the economics of information and incentives of the last 30 years has shown, perfect intragenerational insurance markets constitute a rather heroic assumption. It is nevertheless a necessary assumption here, because we want to concentrate on macroeconomic risks. Aggregate risk can be classified into productivity risk, valuation risks, demographic risks, and political risks.

4.2.1 Productivity Risk and Valuation Risks

Stochastic Total Factor Productivity, Wage Rate, and Interest Rate. The most commonly analyzed risk is that of stochastic total factor productivity (TFP). A common specification in modelling aggregate risk is to include a stochastic productivity term in a production function that is familiar from the real business cycle and neoclassical growth literature where the inputs to production are capital (K), labor (N), and "technology" (A). For illustrative purposes we assume that the production function is Cobb-Douglas and that the technological progress is labor-augmenting.[5] Thus, output (Y_t) in period t is equal to:

$$Y_t = K_t^{\alpha}(A_t N_t)^{1-\alpha}, \quad 0 < \alpha < 1. \tag{4.2}$$

In competitive markets, labor and capital are paid at their marginal product. The real wage rate (w_t) and the return of capital before depreciation (r_t) are thus

$$w_t = (1-\alpha)A_t^{1-\alpha}K_t^{\alpha}N_t^{-\alpha}, \text{ and} \tag{4.3}$$
$$r_t^k = \alpha K_t^{\alpha-1}(A_t N_t)^{1-\alpha}, \tag{4.4}$$

respectively.

The elasticities of both factor prices with respect to the technological shock, $\xi_{wA} \equiv \frac{\partial w_t}{\partial A_t} \cdot \frac{A_t}{w_t}$ and $\xi_{rA} \equiv \frac{\partial r_t}{\partial A_t} \cdot \frac{A_t}{r_t}$ are therefore equal to $1-\alpha$, respectively.

[5] See Barro and Sala-i-Martin (1995, pp 32–36) for a discussion of different types of technological progress.

Thus, the wage rate and interest rate are both exposed to productivity risks in a very similar fashion. Empirical evidence in Baxter and Jermann (1997) and Bohn (1999) also shows that over long periods, e.g. 30 years, the correlation of labor and capital income is close to one.

Valuation Risks: Stochastic Depreciation. In the paragraph above we have argued that the wage rate and the interest rate should exhibit a high positive correlation because both are affected by stochastic total factor productivity. It is nevertheless reasonable to assume, that the risk characteristics are not identical. Bohn (1998,1999) has modelled this in an elegant way by introducing a stochastic valuation for old capital.

Total resources available in the economy are given by aggregate output plus old capital evaluated at its current value: $Y_t + v_t K_t$, where v_t is value of old capital in period t. Note that $(1 - v_t)$ can be interpreted as a stochastic depreciation rate for existing capital. Stochastic depreciation may for example result from specific advances in a technology that lead to an unforseen, faster depreciation of existing capital.[6] The gross return to capital in period t is equal to the marginal return of capital in period t plus the value of old capital in period t:

$$R_t^k = \alpha K_t^{\alpha-1}(A_t N_t)^{1-\alpha} + v_t. \tag{4.5}$$

The elasticity of the gross return with respect to productivity shocks is given by $\xi_{RA} \equiv \frac{\partial R_t^k}{\partial A_t} \cdot \frac{A_t}{R_t^k} = (1-\alpha)\frac{R_t^k - v}{R_t^k}$. Since v is positive, $\xi_{RA} < \xi_{rA} = \xi_{wA}$. Additionally, valuation risks have to be considered for the gross return to capital. The elasticity of R_t^k with respect to the valuation term is equal to $\xi_{Rv} \equiv \frac{\partial R_t^k}{\partial v_t} \cdot \frac{v_t}{R_t^k} = \frac{v}{R^k}$. Obviously, v_t does not have an impact on the wage rate. Therefore, only the old generation is exposed to the valuation risk. Bohn (1999) argues that a social security trust fund that invests in equity is a tool to generate intergenerational risk sharing of the valuation risk.[7] Importantly, the trust fund that Bohn has in mind is not comparable to equity investments via individual accounts. Under an individual account scheme, the old generation still bear the entire valuation risk. Social security with a trust fund that is partially invested in risky equity makes future contribution payments, rather than benefits, contingent on the performance of capital. Thus, parts of the asset price uncertainty can be effectively shifted away from the old generation towards the young generation.[8]

---

[6] Bohn (1999) further differentiates the shocks to valuation between valuation shocks for equity in aggregate and firm specific shocks which he calls *relative return risk*. His empirical results indicate that this latter effect is not of relevance. Therefore we do not include this expansion.

[7] Note that capital income taxes may serve as an alternative instrument to share valuation risk across generations; see Smetters (2000).

[8] Note that the argument in favor of the trust fund investing in equity is not the equity premium. It would be a fallacy to simply use the equity risk premium as an argument for equity investments without taking account of the risk implications and the associated costs; cf. Sect. 7.2.2.

Asset Bubbles. For completeness, we also touch upon the issue of asset bubbles. Asset bubbles and other anomalies, in particular, the excess volatility in stock prices, have sparked a move in the theoretical research of financial markets "from efficient markets theory to behavioral finance"; see e.g. Shiller (1999a,2003a).[9] As the developments of stock markets during the past five to ten years throughout the world has once again shown, asset bubbles certainly represent a huge risk on an individual level for both households and firms. However, at the macroeconomic level (of a closed economy), the primary effects of an asset bubble is a zero-sum game, i.e. when adding up realized gains and losses over the cycle of an asset bubble and its burst, there are as many losers as there are winners. Generally, sufficient diversification should by and large eliminate unsystematic risk. However, systematic risk may nevertheless prevail. The costly effect of an asset bubble at the macroeconomic level are due to the allocative inefficiency that may arise from the bubble, such as distorted decisions on the household level due to wealth effects and distorted investment decisions. Further, financial crises may raise the cost of financial intermediation and restrict credit. As a result, the level of activity of the real sector may be restrained which may eventually lead to period of low growth.[10]

4.2.2 Demographic Risk

Neglecting migration, the size of the demographic process can be fully described by age-specific fertility rates and survival rates. For precise population projections, as given in Sect. 3.1, a demographic model must account for age-specific mortality rates and fertility rates at an annual interval. Here, we are more concerned with changes to age groups that are important in a macroeconomic sense. For ease of notation and simplicity in the presentation we will restrict the economic analysis to the most simple model with only two periods as it has been introduced in Chap. 2. However, in order to give a more intuitive understanding of the demographic process itself, we will first explain the demographic process within a three-period model.

Specifically, at each point in time t the population is composed of three generations: the young (N_t^y), the middle-aged (N_t^m), and the elderly (N_t^o). The size of the young cohort is determined by the size of the cohort of the middle-aged times the population growth rate. To take life-length uncertainty into consideration we introduce a survival-probability between the second and the last period of the life-cycle. The old generation is thus determined by the size of the middle aged cohort in period $t-1$ (i.e. the generation born and young in $t-2$) and the survival probability. In order to model a stochastic demographic process, we split up the fertility rate and the survival rate in a deterministic and a random component, respectively. Hence, the population growth is given by $1 + n + \hat{\eta}_t$ and the survival probability is given by $p + \hat{\pi}$, where n and p are the deterministic components and $\hat{\eta}$ and $\hat{\pi}$ the stochastic components, respectively. The three cohorts alive in period t can then be modeled by Eq. (4.6).

[9] As has been discussed in Sect. 2.3.3, asset bubbles may be part of a rational expectations equilibrium. However, only positive asset bubbles can be explained within this framework.
[10] See e.g. Allen and Gale (1998) on financial crises.

Table 4.1. Relative size of cohorts in period t

in relation to \ size of cohort	N_t^y	N_t^m	N_t^o
N_t^y	1	$\frac{1}{1+n+\hat{\eta}_t}$	$\frac{p+\hat{\pi}_t}{(1+n+\hat{\eta}_t)(1+n+\hat{\eta}_{t-1})}$
N_t^m	$(1+n+\hat{\eta}_t)$	1	$\frac{p+\hat{\pi}_t}{1+n+\hat{\eta}_{t-1}}$
N_t^o	$\frac{(1+n+\hat{\eta}_t)(1+n+\hat{\eta}_{t-1})}{p+\hat{\pi}_t}$	$\frac{1+n+\hat{\eta}_{t-1}}{p+\hat{\pi}_t}$	1

$$\begin{aligned} N_t^y &= (1+n+\hat{\eta}_t)N_t^m \\ N_t^m &= N_{t-1}^y = (1+n+\hat{\eta}_{t-1})N_{t-1}^m \\ N_t^o &= (p+\hat{\pi}_t)L_{t-1}^m = (p+\hat{\pi}_t)L_{t-2}^y = (p+\hat{\pi}_t)(1+n+\hat{\eta}_{t-2})N_{t-2}^m . \end{aligned} \tag{4.6}$$

For economic analysis, the relative size between the different generations is much more interesting than the absolute size of the population. The reason for this is that economic variables are only meaningful in per-capita terms. Also, demographic pressure of an aging society is characterized by the development of the the relation between retired and working population, i.e. the old-age dependency ratio that was used as a statistic in order to describe the demographic transition in Chap. 3. For this reason, it is useful to express the size of the different generations in relation to the size of another generation. Usually the normalization is chosen such that variables are expressed as a relation to the size of the current working generation. In Table 4.1, the relative sizes of the different generations alive in t are expressed in their relative size to each other.

The presentation within a three-period model of overlapping generations was mainly chosen in order to give the reader a more intuitive understanding of the demographic process.[11] For a two-generation setup, one can drop the young generation (N_t^y) in Table 4.1 and use the relative size between the working aged population and the old generation. There are some advantages to choosing a three generation setup. First, the timing of fertility shock is much more realistic in a three-generations setup. Changes to birth rate will only affect the working population after roughly 25 years. Second, in a two-generation setup where $p = 1$, the length of the retirement period is overstated. Third, a model with more than two periods will feature private risk shar-

[11]The stochastics of the demographic process can also be modelled in a much more sophisticated way. Ríos-Rull (2001) has put forward a stochastic ARIMA Leslie-Matrix and a solution-method for this procedure. Also, the uncertainty underlying population projections for medium to long term horizons has been addressed. Lee and Tuljapurkar (2001) have developed confidence intervals for population projections. This helps to put population projections as they have been presented in Chap. 3.1 on a statistical sound foundation.

ing between the middle-aged and old agents.[12] However, private risk sharing with the unborn generation will still remain impossible. Fourthly, models with a small number of overlapping generations aggregate a long time span into one single observation. The influence of single shocks will therefore be over-amplified. In real life, the *law of large numbers* will more likely come to application than when restricting the perspective to a life that consists of two realizations from a random distribution. Finally, the exposure to a single shock is also overstated because of the strong influence one realization may have on the economic variables of the next period. For the sake of a tractable analysis, we will nevertheless use the two-generations setup in the preceding chapters.

Risks Resulting from Demographic Uncertainty. Demographic risks affect life-cycle resources via factor price changes and via social security. The elasticities of factor prices can be computed from Eq. (4.3). The elasticities are given by:

$$\xi_{wN} \equiv \frac{\partial w_t}{\partial N_t^m} \cdot \frac{N_t^m}{w_t} = -\alpha, \text{ and} \tag{4.7}$$

$$\xi_{RN} \equiv \frac{\partial R^k}{\partial N_t^m} \cdot \frac{N_t^m}{R^k} = \alpha, \tag{4.8}$$

respectively.

On an intuitive level, the elasticities of the wage rate and the return of capital with respect to changes in the size of the working population have already been illustrated in Sect. 3.2.2. Accordingly, wage income responds negatively to positive shocks to the size of the working population, whereas the influence of the same shock is positive for capital income. In fact, the elasticities are of equal size, but of opposite sign. The argument is based on the changes to relative scarcity of the input factors. Contrary to fertility shocks, longevity shocks do not have an influence on factor incomes.

Next to the factor income effects, PAYG social security needs to adjust either the contribution rate, or benefits, or both in response to demographic shocks. To illustrate this we depart from the formula for wage-indexed social security with demographic indexation given in Fig. 2.1:

$$\beta_t = \bar{\gamma}\, w_t \left[(1-\varrho) + \varrho \frac{\mathrm{E}[OAD_t]}{OAD_t}\right] = \bar{\gamma}\, w_t \left[(1-\varrho) + \varrho \frac{p}{1+n}\frac{1+n+\hat{\eta}}{p+\hat{\pi}}\right]. \tag{4.9}$$

Linearizing Eq. (4.9) in levels yield:

$$\tilde{\beta} = \tilde{w} + \varrho[\tilde{\eta} - \tilde{\pi}], \tag{4.10}$$

where $\tilde{x} \equiv \frac{dx}{x}$ around the steady state levels of x. The elasticities of the benefit payments with respect to the different variables are given by the 1, ϱ, and $-\varrho$ for $\hat{w}$, $\hat{\eta}$, and $\hat{\pi}$, respectively. Not surprisingly, the elasticity of benefits with respect to

[12] A similar argument is commonly made against the possibility of Diamond's (1965) over-accumulation in the deterministic setup.

wage income is unity, and the elasticity of demographic changes is equal to the policy parameter ϱ with a positive sign for fertility shocks and a negative sign for longevity shocks. The elasticities of the contribution rate equal $(1-\varrho)$ and $-(1-\varrho)$ for relative changes of η and π, respectively. In the real world, pension schemes are usually not specifically indexed to demographics.[13] Instead, most benefit schemes fully adjust to longevity, i.e. $\varrho_\pi = 0$. The treatment of fertility shocks is often not specified. This will leave room for discretionary political adjustments. In a sense, this can be interpreted as a political risk.

4.2.3 Political Risk

When speaking of political risk, we refer to the possibility that today's laws concerning public tax-transfer programs may be altered by political decisions.[14] Specifically, we will concentrate on changes concerning the social security program, namely the benefit rule. This may take the form of changes in the formula used to calculate the benefit payments to the old. Within the stylized setting of our discussion, the risk could be portrayed by making the pension policy parameters γ and ρ stochastic. Further changes may be a switch from wage-indexation to inflation-indexation. In fact, the fantasy of the policy maker with respect to changing the generosity of the benefit rule is unlimited. The analysis of Chap. 6 will demonstrate this clearly for the German case.

Other risks that may be classified to be "political risks" are inflation and debt default. The existence of inflation indexed securities, however, provide an effective tool to protect oneself against inflation risk (at least in theory); cf. Chap. 7. While government bonds may be subject to default, this risk can be considered diminishingly small in the more developed countries.

Finally we do not consider any catastrophes, i.e. wars, earthquakes or other events that may completely destroy the capital stock of a region. The benefits from intergenerational, but also international risk sharing are obvious for these drastic scenarios.

4.3 Overview

We have given a broad classification of different source of aggregate risk above. In Table 4.2, we summarize the risky components of a representative individual's life-cycle resources. It is convenient to write the budget constraint in present value terms of the second period. By choosing this presentation, one can eliminate some confusion

[13]The benefit formula of Germany was intended to be indexed to longevity in 1999. However, the formula never came to application. A new law may reintroduce a demographic component in the benefit formula. See also Chap. 6.

[14]We have concentrated purely on macroeconomic risk. Note that the risk we call political risk may also be interpreted as an aggregate risk on a generational level.

Table 4.2. Risky components of life-cycle resources in a closed economy

$c_t^y(1+r)+c_{t+1}^o \ =$	$\hat{w}_t$	$\cdot\,(1-\tau_t^y)\cdot$	$[\omega(\hat{r}_{t+1}$	$+\,\hat{v}_{t+1})$	$+\,(1-\omega)(1+r^f)]$	$+\,\beta_{t+1}$
possibly affected variable by...	$\hat{w}_t$	τ_t^y	$\hat{r}_{t+1}$	$\hat{v}_{t+1}$		β_{t+1}
demographic risk	x	x	x			x
productivity risk	x	x	x			x
valuation risk				x		
political risk		x				x

that comes along with discounting when the interest rate is partially stochastic.[15] Life-cycle resources consist of the labor income after social security contributions times the composite gross interest rate. This composite gross interest rate is the combination of the gross returns on risky assets ($\hat{r}+\hat{v}$) and risk-free assets (r^f), where the fraction of savings invested in risky assets equals ω. We define risk-free assets as government bonds that yield constant r^f. In addition, the individual receives benefit payments from the public pension scheme β_{t+1} in the old period.

From an *ex-ante* perspective, demographic risks may have an influence on the wage rate and the interest rate due to changes in the capital-labor ratio. Depending on the specific design of social security, either contributions, benefits, or both will adjust to demographic fluctuations.[16] Stochastic total factor productivity also represents a risky component for labor income and income from risky assets. Depending on the policy of the public pension program, either benefits or contributions or both are affected by shocks to TFP. Stochastic depreciation adds another source of uncertainty that is captured within the gross return to capital. Note that only the old generation is exposed to this type of risk. This is also true for the case with PAYG social security. Only if parts of the social security program's trust fund is invested in capital will this risk be partially transferred to the young generation.

In the preceding chapters, we will address some very specific issues of social security under uncertainty. The coverage is by far non-exhaustive but provide insights on some issues at hand. Specifically, our focus lies on the interaction of demographic risks with other sources of risk. First, we will expand the simple examples of Sect. 4.1 by stochastic demography in Chap. 5. We thus analyze the risk sharing characteristics of social security when labor income and demographics are uncertain. In Chap. 6, we

[15] We compound the consumption of the young period on the left hand side by an unspecified interest rate r. Since we are interested in the RHS of the budget constraint, namely the life-cycle resources, we don't bother in clarifying the interest rate on the LHS. But for the identity to hold, $1+r$ should equal the term in squared brackets on the LHS. As an alternative representation, it is sometimes assumed that consumption only occurs in the second period.

[16] Not included in Table 4.2 are the effects of the aforementioned "asset meltdown hypotheses"; see Sect. 3.2.2.

take a closer look at the political risk of benefit rule changes. For the example of the German case study, we construct an indicator to measure the political changes of the generosity of the benefit formula over the past three decades. Furthermore, we argue that our empirical results point to a close link between demographic developments and the generosity of the benefit formula. Finally, in Chap. 7 we address the issue that social security may also be beneficial in terms of being an instrument for risk diversification. If social security and returns to risky investments are negatively correlated, social security may serve as a hedging tool for optimal portfolio choice.

Chapter 5
Labor Income Risk, Demographic Risk, and the Design of (Wage-Indexed) Social Security

The intergenerational risk sharing characteristics of social security are a crucial argument in the defense of pay-as-you-go (PAYG) pension programs in the ongoing debate on privatizing social security.[1] With overlapping generations, a market solution to share risks between generations is not feasible. Enders and Lapan (1982) argue that social security can be a substitute for this missing insurance market. Thøgersen (1998) has shown that for stochastic labor income, the design of pension programs plays a crucial role: only PAYG pension programs with wage indexation are capable of sharing risks.

In this chapter, we point out that the gains of sharing labor income risks across generations are, however, bought at the cost of exposing life-cycle resources to risks associated with the uncertainty of the population growth rate. In a small open economy, this demographic uncertainty would not be present without social security.[2] Generally, under the presence of social security, population growth uncertainty may have an impact on the risk exposure of life-cycle resources via two channels: First, the relation between contributions and benefits is affected by deviations from the steady state old-age dependency ratio. Therefore, the return on the contributions paid into the pension program is uncertain. Second, the wage rate may change depending on cohort size. While the first effect will always occur, the second effect depends on whether the fertility shock will have an affect on the wage rate, as it is predicted by neoclassical growth theory for a closed economy. We concentrate on how uncertainty concerning population growth influences the variance of life-cycle resources from an *ex ante* point of view.[3]

The question addressed in this chapter is not how to respond to a baby-boom baby-bust scenario, but how a PAYG pension scheme transmits demographic risk onto the life-cycle resources of the individuals. For this purpose, we will add a stochastic population growth rate to the simple two-period overlapping generations framework

[1] This Chapter draws extensively on Borgmann (2002).

[2] We use the term *small open economy* for a partial equilibrium model where wage and interest rates are exogenous. As a counterpart, we will present a partial equilibrium model with endogenous wage rate, but exogenous interest rate. We will be semantically imprecise and refer to this model as a *closed economy*. This terminology is rather euphemistic for the ad-hoc approach we use. We nevertheless use the term in order to have a catchy name for this scenario.

[3] See Rangel and Zeckhauser (2001) on *ex-ante* versus *interim* optimality.

introduced by Gordon and Varian (1988) and applied to the design of social security with stochastic labor income by Thøgersen (1998).

Furthermore, we will restrict the analysis to pension schemes with wage-indexation, as only these are capable of intergenerational risk sharing. However, under stochastic population growth, wage-indexation does not fully describe the design of the pension program. Instead, one needs to introduce policies on how the pension scheme reacts to a demographic shock; cf. Fig. 2.1. We call these different possible pension policies *defined contribution wage indexation* (DC) and *defined benefit wage indexation* (DB). The basic difference between the two schemes is whether contributions (former) or benefits (latter) are adjusted in response to the realization of the uncertain population growth rate.

In the next section, we present the model and the two pension policies. In Sect. 5.2, the risk sharing properties of the policy schemes are analyzed in a small open economy. In order to capture the general equilibrium effects of demographic changes on labor income that should occur in a closed economy, we allow for the wage rate to change in response to fertility shocks in Sect. 5.3. In Sect. 5.4, we introduce a hybrid policy concerning the demographic indexation of the pension scheme and derive optimal demographic indexation rules for the small open and the closed economy. The interpretation of demographic shocks is expanded to also cover longevity shocks in Sect. 5.4. Section 5.6 summarizes.

5.1 A Simple Overlapping Generations Model with Stochastic Labor Income and Stochastic Population Growth

We model risky labor income in a two period overlapping generations economy with a stochastic population growth rate under the presence of a PAYG pension scheme. In each period t there are two generations alive. The young generation inelastically supplies one unit of labor. The stochastic gross labor income is described by:

$$w_t = w \cdot \varepsilon_t, \tag{5.1}$$

where w is deterministic and ε_t is a normal independent identically distributed (n.i.i.d.) stochastic variable with mean one and variance σ_ε^2.[4] From this gross labor income, the individual will need to finance his youth consumption, a contribution to the pension scheme (τ_t), and further private savings for retirement. The real rate of interest (r) is exogenous, positive, and by assumption constant over time. When old, the individual does not work but consumes the gross return of his savings plus the social security transfer (β_{t+1}). There is neither lifetime-uncertainty nor any bequest motive. The present value of life-cycle resources at birth (y_t) of a representative individual born in period t equal:

$$y_t = w_t - \tau_t + \frac{\beta_{t+1}}{1+r}. \tag{5.2}$$

[4]Note that ε is a multiplicative term, whereas ϵ in Sect. 4.1 is an additive shock.

In order to capture risk aversion of the individuals in a simplified setting, we follow Gordon and Varian (1988) and Thøgersen (1998) and assume a mean-variance utility function, where utility increases with the expected present value of life-cycle resources but decreases with its variance:

$$U_t = u(\mathrm{E}[y_t]) - v(\mathrm{Var}[y_t]). \tag{5.3}$$

As a second source of uncertainty, we add a stochastic component $\hat{\eta}_t$ to the population growth rate. Since we are interested in the *ex-ante* risk implications of social security, we do not model a specific baby–boom baby–bust scenario. Instead, we introduce an additive stochastic component to the otherwise constant population growth rate. The demographic process is described by:

$$N_{t+1} = (1 + n + \hat{\eta}_t)N_t, \tag{5.4}$$

where n is the deterministic part of the growth rate and $\hat{\eta}_t$ is a n.i.i.d. stochastic variable with mean zero and variance σ_η^2.[5]

For PAYG social security, we will only consider pension schemes that are indexed to labor income such as the *fixed tax rate* case of Thøgersen (1998); see left branch in Fig. 2.1. However, we will differentiate between a defined contribution and a defined benefit wage indexation.[6] We speak of a defined contribution system, where the contribution payments of the young generation are a fixed share γ of their income. The budget of the pension program is assumed to be balanced every period without running deficits or surpluses. Per-capita contribution and transfer payments for a representative member of generation t are therefore given by:

$$DC: \quad \tau_t = \gamma[w\hat{\varepsilon}_t] \quad \text{and} \quad \beta_{t+1} = \gamma(1 + n + \hat{\eta}_t)[w\hat{\varepsilon}_{t+1}]. \tag{5.5}$$

In contrast, we speak of a defined benefit scheme when the retirees obtain a fixed share (replacement rate) ψ of the per-young-capita labor income. For this case, the contribution payment will vary with population growth in order to guarantee the promised pension payment. The respective per-capita contribution and transfer payments are then:

$$DB: \quad \tau_t = \frac{\psi}{1 + n + \hat{\eta}_{t-1}} w\hat{\varepsilon}_t \quad \text{and} \quad \beta_{t+1} = \psi w\hat{\varepsilon}_{t+1}. \tag{5.6}$$

[5] A normal distribution is somewhat problematic since it does not guarantee, that $(1+n+\hat{\eta}) > 0$ and $w\varepsilon > 0$. Although a truncated normal could guarantee these conditions, an analytical derivation of the variance using such a distribution would be close to impossible. Restricting the variances of $\hat{\varepsilon}$ and $\hat{\eta}$ to values well below one will at least make such an event highly unlikely.

[6] Most authors do not look at a defined benefit wage indexation, but at a defined benefit scheme that is independent of the next generations income. See Hassler and Lindbeck (1998) for an intergenerational risk sharing analysis of a fixed tax rate with indexation versus a defined benefit scheme without indexation. The distinction between DC and DB as discussed here can – to the authors knowledge – only be found in Bohn (2001).

The difference between the two schemes lies in the exposure to the demographic shock: while in the defined benefit scheme, the income of the old generation is independent of the realization of the demographic shock $\hat{\eta}_t$, it is dependent on $\hat{\eta}_t$ in the defined contribution scheme. However, the respective contribution rates react just in the opposite way: contribution payments of the young generation are adjusted under the defined benefit scheme, while they are held constant under defined contribution. Over the life-cycle, the two schemes differ in respect to whether generation t is exposed to the demographic stochastic variable of period t (defined contribution) or to the demographic shock of period $t-1$ (defined benefit). Under both social security schemes, the life-cycle resources (y_t) of the generation born in period t are subject to three realizations of stochastic variables: the realizations of the stochastic productivity term $\hat{\varepsilon}$ in t and $t+1$ and one realization of the stochastic population growth component $\hat{\eta}$. we will now analyze the risk sharing characteristics of the different social security schemes.

5.2 PAYG Pension Programs in a Small Open Economy with Stochastic Labor Income and Population Growth

In this section, we focus the attention on a small open economy. In a small open economy, domestic per-capita capital stock is independent of domestic savings. This assumption together with perfect competition on factor markets assures that wage and interest rates will be constant. Therefore, macroeconomic feedback effects on factor prices are not present.

5.2.1 Defined Contribution Income Indexation

We start out with the defined contribution scheme. In order to calculate the mean and variance of life-cycle resources, we substitute τ_t and β_{t+1} from Eq. (5.5) and $w_t = w\hat{\varepsilon}_t$ into Eq. (5.2):

$$y_t^{DC} = w\hat{\varepsilon}_t(1-\gamma) + \frac{\gamma w}{1+r}\hat{\varepsilon}_{t+1}(1+n+\hat{\eta}_t). \tag{5.7}$$

Because $\hat{\varepsilon}_{t+1}$ and $\hat{\eta}_t$ are independent, the expectation of Eq. (5.7) is equal to $\mathrm{E}[y^{DC}] = w - \gamma w\frac{r-n}{1+r}$. In comparison to a purely funded scheme, where $\tau = \beta = 0$, the expectation of life-cycle resources $\mathrm{E}[y_t]$ will be smaller (larger) under PAYG social security than under a funded system when the economy is on a dynamically efficient (dynamically inefficient) growth path.

The variance of y^{DC} is given by:[7]

[7]The variance is derived in Appendix A.1 for the more general case with macroeconomic feedback effects discussed in Sect. 5.3. Equation (5.8) is a special case of Eq. (5.19) with $\alpha = 0$.

$$\begin{aligned}\mathrm{Var}[y^{DC}] = w^2 \Bigg\{ & \left[(1-\gamma)^2 + \gamma^2 \left(\frac{1+n}{1+r} \right)^2 \right] \sigma_\varepsilon^2 \\ & + \left(\frac{\gamma}{1+r} \right)^2 \sigma_\eta^2 + \left(\frac{\gamma}{1+r} \right)^2 \sigma_\varepsilon^2 \sigma_\eta^2 \Bigg\}. \end{aligned} \tag{5.8}$$

The term on the first line on the RHS of Eq. (5.8) is the *Thøgersen case* for a multiplicative labor income shock under deterministic population growth ($\sigma_\eta^2 = 0$). An indexed pension scheme with contribution rate $\gamma > 0$ reduces the variance of life-cycle resources in comparison to a fully funded scheme ($\gamma = 0$), where the variance is equal to $w\sigma_\varepsilon^2$. The second line shows the effect of stochastic population growth ($\sigma_\eta^2 > 0$): the *demographic-risk-effect* displays how much the variance of life-cycle resources rises due to demographic uncertainty added by the pension scheme. The first term of this line is the pure demographic risk effect while the second term is a combined uncertainty of both population growth and labor income. Both terms on line two are obviously positive for $\gamma > 0$ and therefore lead to an increase of the variance. Since the demographic shock enters life-cycle resources in the second period, both terms on line two are discounted by $(1+r)^{-2}$. Note that line two is zero for an economy with deterministic population growth. The deviation of both lines together from $w\sigma_\varepsilon^2$ represent the total risk-impact of wage-indexed social security with stochastic population growth and stochastic labor income in a small open economy. As one can see, the variance unambiguously increases in comparison to the *Thøgersen case*. This leads immediately to the most important result: one cannot share labor income risk without being exposed to a demographic risk.

We analyze optimal policy in a sense that the optimal choice of γ is defined as the value of the tax rate γ^* that minimizes the variance of y_t^{DC} ($\gamma^* \equiv \underset{\gamma}{\mathrm{argmin}}\{\mathrm{Var}[y_t^{DC}]\}$). Differentiating Eq. (5.8) with respect to γ and solving for γ^* yields:

$$\gamma^* = \frac{1}{1 + \left(\frac{1+n}{1+r} \right)^2 + \frac{\sigma_\eta^2 + \sigma_\eta^2/\sigma_\varepsilon^2}{(1+r)^2}}. \tag{5.9}$$

In the golden rule steady state ($r = n$) we have $\gamma^* = \frac{1}{2+(\sigma_\eta^2+\sigma_\eta^2/\sigma_\varepsilon^2)(1+r)^{-2}}$, so that the optimal value of the tax rate for this special case is smaller than $\frac{1}{2}$, which would equal the Thøgersen solution for $r = n$.[8] The power of social security to share risk amongst generations is reduced when population growth is stochastic. Obviously, the optimal tax rate will have to be lower. How large this reduction will be, depends on the interest rate and the variances of the two stochastic variables. A higher interest rate will be associated with a larger optimal tax rate. The same can be said for the variance of labor income. A large variance of population growth will, however, reduce the optimal tax rate. This relationship becomes even stronger, if the variance of population growth

[8]The Thøgersen solution is obtained by setting σ_η in Eq. (5.9) equal to zero: $\gamma^*_{\text{Thøgersen}} = \frac{1}{1+(\frac{1+n}{1+r})^2}$.

is large in relation to the variance of labor income. Again, this is no surprise: labor income shocks can only be shared between generations when life-cycle income is exposed to population risks that are otherwise not present in a small open economy. Hence, when the necessary evil of the remedy (population risk) is large relative to the initial flaw (labor income risk), the gains of a treatment (PAYG social security) are substantially lowered.

5.2.2 Defined Benefit Income Indexation

Proceeding as before, life-cycle resources can be derived by substituting the social security policy rule given in Eq. (5.6) and labor income given in Eq. (5.1) into Eq. (5.2):

$$y_t^{DB} = w\hat{\varepsilon}_t \left(1 - \frac{\psi}{1+n+\hat{\eta}_{t-1}}\right) + \frac{\psi w}{1+r}\hat{\varepsilon}_{t+1}. \tag{5.10}$$

Since the stochastic variable $\hat{\eta}$ is now part of the denominator, the exact distribution of y^{DB} cannot be determined. However, we can derive some general conclusions about the expectation of y^{DB} and we can approximate both the expectation and variance for our special case with normal distributed population growth. The expectation of life-cycle resources is represented by:

$$\mathrm{E}[y_t^{DB}] = w + \frac{\psi w}{1+r} + \psi w\,\mathrm{E}\left[-\frac{1}{1+n+\hat{\eta}_{t-1}}\right]. \tag{5.11}$$

Because $\frac{-1}{1+n+\hat{\eta}}$ is strictly concave for $\hat{\eta} > -(1+n)$, we have from Jensen's inequality that $\frac{-1}{1+n+\mathrm{E}[\hat{\eta}]} > \mathrm{E}[\frac{-1}{1+n+\hat{\eta}}]$, unless $\hat{\eta}$ is a constant with probability one.[9] Therefore, the expectation of life-cycle resources under defined benefit is smaller for stochastic population growth than for deterministic population growth ($\mathrm{E}[\hat{\eta}] = 0$ with probability one). Note that this is not the case under DC. For a comparison between the two schemes, we define an *equivalent certain benefit level* $\hat{\psi} \equiv \gamma(1+n)$. This is the benefit level that makes the defined benefit scheme equivalent to the defined contribution scheme under deterministic population growth.[10] Substituting this benefit level into Eq. (5.11) yields an important result: for an equivalent certain benefit level the expectation of life-cycle resources under DB is always lower than the expectation of life-cycle resources under DC: $\mathrm{E}[y_{\hat{\psi}}^{DB}] < \mathrm{E}[y^{DC}]$. Note that this result does not depend on the assumed distribution of the random variable $\hat{\eta}$, but holds in general.[11] It is solely due to the fact that under DC, the life-cycle resources are a linear function of the population random variable, while under DB, life-cycle resources are a concave function of the population shock.

[9] Jensen's inequality states that the expectation of non-linear functions evaluated at the random variable is not equal to the function evaluated at the expectation of the random variable.

[10] That the two pension policies react differently to the population shock is not touched by using this specific benefit level. It is only necessary to make the schemes comparable since the parameter determining the size of the pension program, γ and ψ, refer to different generations.

[11] We do need to assume that the expectation over $\hat{\eta}^{-1}$ actually exists.

For our specific case with an assumed normal distribution of $\hat{\eta}$, only an approximate solution can be derived. The quadratic approximation of $(1+n+\hat{\eta})^{-1}$ around $\mathrm{E}[\hat{\eta}]=0$ is given by:

$$\frac{1}{1+n+\hat{\eta}} \approx \overbrace{\underbrace{\frac{1}{1+n} - \left(\frac{1}{1+n}\right)^2 \hat{\eta}}_{\text{linear approximation}} + \left(\frac{1}{1+n}\right)^3 \hat{\eta}^2}^{\text{quadratic approximation}}. \tag{5.12}$$

Substituting the quadratic approximation into life-cycle resources and taking the expectation yields:

$$\mathrm{E}[y^{DB}] \approx w\left(1-\frac{\psi}{1+n}\right)+\frac{\psi w}{1+r}-\psi w\left(\frac{1}{1+n}\right)^3 \sigma_\eta^2. \tag{5.13}$$

The impact of social security on the expectation of life-cycle resources is twofold. First, as always, social security changes life-cycle resources by $\frac{(n-r)\psi w}{(1+n)(1+r)}$ in comparison to a funded system. Second, for the case of defined benefit wage-indexation, the expected value of life-cycle resources is further reduced by $\psi w(1+n)^{-3}\sigma_\eta^2$. The size of this reduction is a positive function of the replacement rate (ψ) and the variance of population growth (σ_η^2), but a negative function of the expected population growth rate $(1+n)$. In comparison, neither a funded system nor the defined contribution wage-indexation are subject to this second effect.

Linear approximation of the variance under defined benefit

We start out to discuss the variance of y^{DB} with the linear approximation of $(1+n+\hat{\eta})^{-1} \approx (1+n)^{-1}-(1+n)^{-2}\hat{\eta}$. Substituting the linear approximation of $(1+n+\hat{\eta})^{-1}$ into the definition of the variance yields Eq. (5.14). We add the superscript ^{la} to denote the linear approximation:

$$\begin{aligned}\mathrm{Var}^{la}[y^{DB}] = & w^2\left\{\left[\left(1-\frac{\psi}{1+n}\right)^2+\left(\frac{\psi}{1+r}\right)^2\right]\sigma_\varepsilon^2\right. \\ & \left.+\frac{\psi^2}{(1+n)^4}\sigma_\eta^2+\frac{\psi^2}{(1+n)^4}\sigma_\eta^2\sigma_\varepsilon^2\right\}.\end{aligned} \tag{5.14}$$

Comparing the linear approximation of the variance for the DB case with the variance of DC at the certain equivalent benefit level ($\hat{\psi}$) shows the similarity of the two. For this benefit level, the respective first line in equations (5.8) and (5.14) is equal, indicating that the pure productivity shock is shared identically within the two schemes. This feature will hold in general, independent of the order of the approximation. Inspection of the second lines of the respective equations shows that there is only one difference between the two schemes: for DB we have a different "discounting mechanism" of the demographic risk than for DC. Remember that under DC, the population risk components in the variance are discounted by the gross interest rate because the shock occurs in the second period. For DB, the exposure to the shock is in the first period and therefore it is not really discounted. The convexity of the function $(1+n+\hat{\eta})^{-1}$ however leads to a "discounting mechanism", where the gross population growth acts as a demographic discount factor. This leads to the result that for $r > n$ (dynamic efficiency), the risk exposure is smaller under DC than under DB. For $n > r$ (dynamic inefficiency), the result is reversed.

Quadratic approximation of the variance under defined benefit

Although easier to handle, using the linear approximation for $(1 + n + \hat{\eta})^{-1}$ may be somehow problematic. In order to ensure that the linear approximation does not overlook important features concerning the variance of y^{DB}, we compare the linear approximation with the quadratic approximation.

The variance of y^{DB} with the quadratic approximation of $(1 + n + \hat{\eta})^{-1}$ given in Eq. (5.12) is derived in Appendix A.2. We adopt the superscript ^{qa} to denote the quadratic approximation. The difference between the linear approximation and the quadratic approximation with $\psi = (1 + n)\gamma$ equals:

$$\begin{aligned} &\mathrm{Var}^{la}[y^{DB}] - \mathrm{Var}^{qa}[y^{DB}] \\ &= w^2 \left[2\frac{\gamma(1-\gamma)}{(1+n)^2}\sigma_\eta^2\sigma_\varepsilon^2 - \frac{\gamma^2}{(1+n)^4}(2\sigma_\eta^2 + 3\sigma_\eta^2\sigma_\varepsilon^2) \right]. \end{aligned} \tag{5.15}$$

This terms will be positive for $[\frac{1-\gamma}{\gamma}(1 + n)^2 - 3/2]\sigma_\varepsilon^2 > 1$. If this condition holds, the linear approximation overestimates the variance and the risk sharing properties under DB are better than indicated above. This will usually be the case for small γ and not too small values for σ_ε^2 and n. For larger γ and small σ_ε^2 and n, the difference given in Eq. (5.15) becomes smaller and the sign may even change.

Unfortunately, the exact results of comparing risk aspects of DC versus DB using the quadratic approximation depends on the parameter values and on the question of whether the economy is in a dynamically efficient region.[12] Specifying parameter values does not seem like a fruitful way to determine a definitive answer to this question, since the model is highly stylized to begin with. Also, a more precise way of modelling risk aversion would be necessary in order to address the issue on how much the individuals are willing to give up in expected income for a reduction of the variance. What this section has however shown, is that a) the expectation of life-cycle resources will be lower under DB than under DC and that b) the discounting mechanism of the demographic shocks in the variance of life-cycle resources differs between the two schemes. All in all, from an ex-ante point of view in a small open economy and for $r > n$, a wage-indexed PAYG pension scheme with defined contribution seems to be superior to a defined benefit scheme.

[12] For $r = n$ (golden rule), the difference given on the RHS of Eq. (5.15) is also equal to the difference of $\mathrm{Var}[y^{DC}_{r=n}] - \mathrm{Var}^{qa}[y^{DB}_{\hat{\psi},r=n}]$. So for the variance to be smaller under DB than under DC, the same conditions apply as for the sign of the difference between the linear and the quadratic approximation.

5.3 Defined Contribution and Defined Benefits in a Closed Economy

In this section, we expand the analysis by making the wage rate dependent on the cohort size of the working generation. Because we want to concentrate on stochastic labor income and population growth, we keep the assumption of a constant interest rate. Even though this procedure is far from modeling a general equilibrium economy, it does however capture the effects of fertility shocks on the wage rate as one would expect to see them in a simple general equilibrium model of a closed economy. Specifically, we assume that the wage rate of period t is given by:

$$w_t = w(\hat{\varepsilon}_t - \alpha\hat{\eta}_{t-1}). \tag{5.16}$$

To see that this specification replicates the macroeconomic effect of population growth on labor income, note that $\hat{\eta}_{t-1}$ is the shock that determines the size of the population born and working in period t. The population N_t is larger for a positive realization of $\hat{\eta}_{t-1}$. From a macroeconomic perspective, one should expect the wage rate to decrease for larger cohort-sizes. The parameter α determines the size of the feedback effect and can be interpreted as the relative change of the wage rate near its steady state value, given an absolute change in population growth of size $\hat{\eta}_{t-1}$ or in other words, as the scaled population-growth-elasticity of the wage rate.[13]

In the remainder of this section, we will first investigate the expectations of life-cycle resources under both schemes. Then we will look at the variance of y^{DC} with $\alpha > 0$ and show how the introduction of the macroeconomic feedback effect changes the results of Sect. 5.2. A comparison of the variance of life-cycle resources under defined contribution and defined benefit concludes the section.

5.3.1 Life-Cycle Resources under Defined Contribution and Defined Benefit

By substituting the contribution and benefit payments given in equations (5.5) and (5.6) for the pension policies and labor income from Eq. (5.16) into Eq. (5.2), one can derive the life-cycle resources under DC and DB respectively:

$$\begin{aligned} y_t^{DC} &= w(\hat{\varepsilon}_t - \alpha\hat{\eta}_{t-1})(1-\gamma) + \frac{w\gamma}{1+r}(\hat{\varepsilon}_{t+1} - \alpha\hat{\eta}_t)(1+n+\hat{\eta}_t), \\ y_t^{DB} &= w(\hat{\varepsilon}_t - \alpha\hat{\eta}_{t-1})\left(1 - \frac{\psi}{1+n+\hat{\eta}_{t-1}}\right) + \frac{w\psi}{1+r}(\hat{\varepsilon}_{t+1} - \alpha\hat{\eta}_t). \end{aligned} \tag{5.17}$$

Taking the expectation of life-cycle resources' yields:

$$\begin{aligned} \mathrm{E}[y^{DC}] &= w(1-\gamma) + w\gamma\frac{1+n}{1+r} - w\alpha\frac{\gamma}{1+r}\sigma_\eta^2, \\ \mathrm{E}[y^{DB}] &\approx w(1 - \frac{\psi}{1+n}) + \frac{w\psi}{1+r} - \frac{w\psi}{(1+n)^3}\sigma_\eta^2 - w\alpha\frac{\psi}{(1+n)^2}\sigma_\eta^2. \end{aligned} \tag{5.18}$$

[13] Note that α used here does not equal the factor share of capital from Eq. (4.2). Instead, $\alpha \equiv \xi_{wN}/(1+n)$; cf. Sect. 4.2.2.

In comparison to the small open economy (Sect. 5.2), the expectations for both policies are reduced by the respective last term. The size of this term depends on the size of the feedback effect, the size of social security, the variance of population growth, and a "discount" factor. Again, the "discounting" of the terms associated with σ_η^2 differs: for equivalent certain benefit level $\hat{\psi}$, the discount factor is $(1+n)^{-1}$ for DB, while it equals $(1+r)^{-1}$ for DC.

5.3.2 Variance of Life-Cycle Resources under Defined Contribution

The variance of life-cycle resources under DC with macroeconomic feedback is given in Eq. (5.19) (see Appendix A.1):

$$\begin{aligned}\mathrm{Var}[y^{DC}] = w^2 \Bigg\{ & \left[(1-\gamma)^2 + \gamma^2\left(\frac{1+n}{1+r}\right)^2\right](\sigma_\varepsilon^2 + \alpha^2\sigma_n^2) \\ & + \left(\frac{\gamma}{1+r}\right)^2 (1+\sigma_\varepsilon^2)\sigma_\eta^2 \\ & + 2\left(\frac{\gamma}{1+r}\right)^2 \alpha^2\sigma_\eta^4 - 2\left(\frac{\gamma}{1+r}\right)^2 (1+n)\alpha\sigma_\eta^2 \Bigg\}. \end{aligned} \tag{5.19}$$

There are important differences for the variance under DC in the closed economy in comparison to the small open economy: the term in square brackets on the first line of Eq. (5.19) is now multiplied by the variance of wage income (σ_ε^2) plus the variance of population growth (σ_η^2) times the coefficient for the macroeconomic feedback squared (α^2). This indicates that in a closed economy, uncertain population growth will already have an impact on the variance of life-cycle resources in a fully funded system. In particular, the variance of life-cycle resources in a fully funded system equals the variance of labor income: $\mathrm{Var}[w_t] = (\sigma_\varepsilon^2 + \alpha^2\sigma_n^2)w^2$ (Eq. (5.19) with $\gamma = 0$). Because w_t is dependent on demographics, labor income is exposed to the risk of uncertain demographic developments. Therefore, social security reduces the risk of fluctuating labor income due to both the productivity shock and the demographic shock. So in the closed economy, the benefits of sharing the risk of fluctuating wage income over generations are greater than in the small open economy. However, the risk of fluctuating wage income due to fertility shocks cannot be shared as easily as the productivity uncertainty. The necessary evil of fluctuating benefit payments adds risk to life-cycle resources. Specifically, the increase of the variance is of size $w^2(\frac{\gamma}{1+r})^2(1+\sigma_\varepsilon^2)\sigma_\eta^2$. This effect is identical for the small open economy and the closed economy.

The second difference between the small open economy and the closed economy is observable in the last line of Eq. (5.19). This line can be interpreted as a "covariance" between wage income and the replacement rate in period $t+1$: the same shock $\hat{\eta}_t$ has an influence on both the replacement rate in $t+1$ and the wage income of $t+1$. Since the two effects are of opposite directions, this "covariance" is negative.[14]

[14] To verify this, note that for plausible values $1+n > \alpha\sigma_\eta^2$, because α and σ_η^2 both should be well below one, so that n must only be not too much below zero to guarantee this condition.

The argument that a PAYG pension program helps to share risks between generations in an economy where demographic fluctuations have a strong impact on factor prices has been made in defense of sustaining a PAYG scheme during a baby-bust scenario by Smith (1982) and Bütler and Harms (2001): while the PAYG scheme is per se exposed to demographic risks, the welfare of the different generations during this demographic transition is affected inversely by macroeconomic effects on factor income. Bohn (2001) and Young (2001) also come to the conclusion that large generations are usually hit hardest because of general equilibrium effects on factor returns; see also Sect. 3.2.2.

5.3.3 Defined Benefit versus Defined Contribution

For the sake of a tractable representation, we will only discuss the variance of life-cycle resources under DB using the linear approximation for $(1+n+\hat{\eta}_{t-1})^{-1}$. The variance of y^{DB} using the quadratic approximation is given in Appendix A.3. Also, we will concentrate on a comparison of DB versus DC, since the general direction of the results are similar for DC and DB. There will be, however, one important new difference between DB and DC when considering macroeconomic feedback on the wage rate: because of the different timing in the pension schemes, DB will offer better insurance for $\hat{\eta}_{t-1}$, while DC offers better insurance for $\hat{\eta}_t$.

Substituting the linear approximation for $(1+n+\hat{\eta})^{-1}$ from Eq. (5.12) into y^{DB} given in Eq. (5.17) yields the following variance:

$$\begin{aligned}\mathrm{Var}^{la}[y^{DB}] =& w^2\left\{\left[\left(1-\frac{\psi}{1+n}\right)^2+\left(\frac{\psi}{1+r}\right)^2\right](\sigma_\varepsilon^2+\alpha^2\sigma_\eta^2)\right.\\ &+\frac{\psi^2}{(1+n)^4}(1+\sigma_\varepsilon^2)\sigma_\eta^2 \\ &\left.+2\frac{\psi^2}{(1+n)^4}\alpha^2\sigma_\eta^4-2\frac{\psi}{(1+n)^2}\left(1-\frac{\psi}{1+n}\right)\alpha\sigma_\eta^2\right\}.\end{aligned} \tag{5.20}$$

The difference between the small open economy and the closed economy under DB is similar to that difference under DC: with the macroeconomic feedback effect, wage income is more risky than without this effect and the gains of insurance via social security are greater (first line). The risk of an uncertain return on the contributions paid into the social security scheme is the same with or without a feedback effect and increases the variance (second line). Finally, as under DC, there is also a "covariance term" that will reduce the variance (third line). However, there is a difference between the respective "covariance terms" for DB and DC.

Since the discounting mechanism of the population shock in the variance still differs between DB and DC, we compare the two schemes in a golden rule steady state with replacement rate $\hat{\psi}$. For this specific case, the variances of the two policies only differ in the last term on the third line. Subtracting the variance of y^{DB} given in Eq. (5.20) with $\psi = \hat{\psi}$ and $r = n$ from the variance of y^{DC} given in Eq. (5.19) yields:

$$\mathrm{Var}[y^{DC}_{r=n}] - \mathrm{Var}^{la}[y^{DB}_{\hat{\psi},r=n}] = \frac{2\gamma}{1+n}(1-2\gamma)\alpha\sigma^2_{\eta}. \tag{5.21}$$

From Eq. (5.21), one can see that for $\gamma < 0.5$, risk sharing is more efficient under DB than under DC.

To clarify this further, we show the influence of positive population shocks in periods $t-1$ and t on the life-cycle resources of generation t under the two policies in a more general setting in Table 5.1. The positive fertility shocks of both periods negatively affect labor income in t and $t+1$, respectively. However, under DC the positive shock $\hat{\eta}_t$ will have a positive influence on the return from the pension program. Under DB, this will be the case for a positive realization of random variable $\hat{\eta}_{t-1}$. The influence of the fertility shock on the return from social security is always of opposite direction from the influence of the same shock on labor income itself.

In general, social security ensures that each generation participates in both fertility shocks and therefore helps to spread risk across generations (first line in equations (5.19) and (5.20)). However, the "covariance term" (third line in equations (5.19) and (5.20)) will only apply for one of the shocks in each scheme: for DC, this is the shock $\hat{\eta}_t$, while for DB it is $\hat{\eta}_{t-1}$. However, $\hat{\eta}_{t-1}$ is the shock that influences the labor income earned by generation t when young, and $(1-\gamma)$ roughly equals the weight of a generation's own labor income in their life-cycle resources. For $\gamma < 0.5$, the influence of a generation's own labor income dominates that generation's life-cycle resources. It follows, that in a realistic setting where $\gamma < 0.5$, DB offers better insurance than DC, because DB provides insurance against fluctuations of the random variable $\hat{\eta}_{t-1}$, which will have a greater weight on total life-cycle resources of generation t.

Bohn (2001) also comes to the conclusion that DB should be preferred to DC. His results are derived in a stochastic dynamic general equilibrium OLG model of a closed economy. His point of departure is different than the one taken in this paper: instead of taking an *ex-ante* perspective, Bohn derives elasticities of how the behavior of the different generations will respond to the realization of the random variables. His results build largely on the general equilibrium effects of the factor prices that will not be present in a small open economy (compare Sect. 5.2). Also, Bohn does not analyze optimal demographic indexation. This will be done in the following section.

All in all, in a closed economy, social security leads to a reduction in the expectation of life-cycle resources that is independent of dynamic efficiency or dynamic inefficiency. At the same time, PAYG pension programs help to insure two risky components of labor income across generations: productivity shocks and fertility shocks. Productivity risk is always reduced via social security. The overall influence of fertility shocks on the variance of life-cycle resources depends in sign and size on the size of the macroeconomic feedback effect and the variance of the population growth rate. Whether social security absorbs or adds risk due to uncertain population growth depends on how strong the effects of demographic changes are on future factor incomes. Studies using computable dynamic general equilibrium models for closed economies predict that demographic risks are reduced by social security; see Sect. 3.2.2.

Table 5.1. Effects of positive fertility shocks on life-cycle resources

DC	$w(\hat{\varepsilon}_t, \hat{\eta}_{-1}t)\,\cdot$	$(1-\gamma)$	$+$	$\frac{\gamma(1+n+\hat{\eta}_t)}{1+r}$	$\cdot\, w(\hat{\varepsilon}_{t+1}, \hat{\eta}_t)$
$\hat{\eta}_{t-1} > 0$	$-$				
$\hat{\eta}_t > 0$				$+$	$-$
DB	$w(\hat{\varepsilon}_t, \hat{\eta}_{t-1})\,\cdot$	$(1-\frac{\psi}{1+n+\hat{\eta}_{t-1}})$	$+$	$\frac{\psi}{1+r}$	$\cdot\, w(\hat{\varepsilon}_{t+1}, \eta_t)$
$\hat{\eta}_{t-1} > 0$	$-$	$+$			
$\hat{\eta}_t > 0$					$-$

5.4 Between Defined Contribution and Defined Benefit: Optimal Demographic Indexation

The discussed cases of defined contribution and defined benefit wage indexation are of course only the polar cases of a continuum of possibilities on how to implement the demographic indexation of PAYG social security.

The burden of the realization of a single population growth shock can be split between the current living young and old generation in any given proportion. In order to capture this in a more general setting, we introduce ρ, defined as the proportion of how much the current old generation's benefits are adjusted in response to the population growth shock.[15] The policy rule for a wage-indexed pension scheme with balanced budgets in every period is then given by:

$$\tau_t = \gamma[w\hat{\varepsilon}_t]\frac{1+n+\rho\hat{\eta}_{t-1}}{1+n+\hat{\eta}_{t-1}} \quad \text{and} \quad \beta_{t+1} = \gamma(1+n+\rho\hat{\eta}_t)[w\hat{\varepsilon}_{t+1}]. \tag{5.22}$$

Note that the earlier discussed cases are special cases of this more general representation, where $\rho = 1$ ($\rho = 0$) equals DC (DB). It seems reasonable to impose the restriction of $0 \leq \rho \leq 1$.

5.4.1 Optimal Demographic Indexation in a Small Open Economy

Life-cycle resources for the more general demographic indexation in a small open economy are obtained by inserting the pension policy from Eq. (5.22) and labor income from Eq. (5.1) into Eq. (5.2):

$$\begin{aligned} y_t &= w\hat{\varepsilon}_t\left[1-\gamma\frac{1+n+\rho\hat{\eta}_{t-1}}{1+n+\hat{\eta}_{t-1}}\right] + \gamma w\hat{\varepsilon}_{t+1}\frac{1+n+\rho\hat{\eta}_t}{1+r} \\ &= w\hat{\varepsilon}_t\left[1-\gamma\left(\rho+\frac{(1-\rho)(1+n)}{1+n+\hat{\eta}_{t-1}}\right)\right] + \gamma w\hat{\varepsilon}_{t+1}\frac{1+n+\rho\hat{\eta}_t}{1+r} \end{aligned} \tag{5.23}$$

The expectation of Eq. (5.23) is given by:

[15] Note that ρ differs in scaling to the policy parameter ϱ used in Fig. 2.1. By using $\rho = \varrho E[\text{OAD}]$, one can derive Eq. (5.22) from Fig. 2.1.

$$\begin{aligned}\mathrm{E}[y_t] &= w - \gamma w \left(\rho + (1-\rho)(1+n)\,\mathrm{E}\left[\frac{1}{1+n+\hat{\eta}_{t-1}} \right] \right) + \gamma w \frac{1+n}{1+r} \\ &\approx w - \gamma w \frac{r-n}{1+r} - (1-\rho)\gamma w \frac{\sigma_\eta^2}{(1+n)^2} \end{aligned} \tag{5.24}$$

The approximate solution given in the second line of Eq. (5.24) is obtained by substituting the quadratic approximation of $(1+n+\hat{\eta}_{t-1})^{-1}$ from Eq. (5.12) into the second line of Eq. (5.24). As before, moving towards a scheme with defined benefit elements ($\rho < 1$) reduces the expectation of life-cycle resources. Because $\mathrm{E}[y_t]$ is strictly increasing in ρ for $\hat{\eta} > -(1+n)$ unless $\mathrm{E}[\hat{\eta}] = 0$ with probability one, the expectation of life-cycle resources is maximized in the corner solution of $\rho = 1$ (DC).[16]

An approximate solution for the variance of life-cycle resources is obtained by substituting the linear approximation of $(1+n+\hat{\eta}_{t-1})^{-1}$ into the second line of Eq. (5.23) and using the familiar definition of the variance:

$$\begin{aligned}\mathrm{Var}^{la}[y] = w^2 \Bigg\{ &\left[(1-\gamma)^2 + \gamma^2 \left(\frac{1+n}{1+r} \right)^2 \right] \sigma_\varepsilon^2 \\ &+ \left[(1-\rho)^2 \left(\frac{\gamma}{1+n} \right)^2 + \rho^2 \left(\frac{\gamma}{1+r} \right)^2 \right] \sigma_\eta^2 (1+\sigma_\varepsilon^2) \Bigg\}. \end{aligned} \tag{5.25}$$

The similarities to Sect. 5.2 are obvious: the variances under DC and DB are identical in the small open economy except for the different "discounting mechanisms" of the terms associated with the variance of population growth. This is again the case here, with ρ determining the weights of the different "discounting mechanisms".

In the polar cases of DC and DB, the terms associated with the variance of the population growth rate are the result of the uncertainty of the realization of the shock in one of the two periods. In a hybrid scheme, where $0 < \rho < 1$, this risk can be reduced because the pension policy allows the individuals to participate in the realization of the demographic shock of both periods. This leads to a reduction of the demographic risk for the individuals in comparison to the two polar cases. The value ρ^* that minimizes the variance of life-cycle resources is given by $\rho^* = \frac{1}{1+\left(\frac{1+n}{1+r}\right)^2}$. For intergenerational risk sharing, neither of the two polar cases is optimal. Instead, a policy that splits the risk that a single demographic shock has on the return of the pension program in roughly equal parts between the living generations is suited best to share the demographic risk. This bears a strong familiarity to the original results concerning intergenerational risk sharing of stochastic labor income under deterministic population growth. Note that ρ^* is independent of γ or the variances

[16] That $\mathrm{E}[y_t]$ is strictly increasing in ρ is proved for the general case in Appendix A.4. For the approximate solution, this can be seen easily by taking the partial derivative of line two in Eq. (5.24) with respect to ρ: $\frac{\partial\,\mathrm{E}[y]^{qa}}{\partial \rho} = \gamma w \frac{\sigma_\eta^2}{(1+n)^2}$.

of the two random variables and will equal 0.5 for $r = n$. For $r > n$, ρ^* will be larger than 0.5 indicating that the optimal policy is closer to DC than DB. This is in line with the result derived in Sect. 5.2: for $r > n$, the difference in the discounting mechanism will favor DC over DB.

Still, even though the demographic risk can be minimized by a pension policy that lies in between DC and DB, one should keep in mind that in the small open economy, the demographic risk would not be present without PAYG social security. Also note that the splitting rule that minimizes the demographic risk will also be subject to an insurance premium, since the expectation of life-cycle resources is reduced for $\rho < 1$ in comparison to a fully funded scheme ($\gamma = 0$) or a pure DC policy ($\rho = 1$). A welfare maximizing policy will therefore depend on the degree of risk aversion of the individuals.

5.4.2 Optimal Demographic Indexation in a Closed Economy

As in Sect. 5.3, we now consider macroeconomic effects of population growth on labor income. Labor income is assumed to behave as specified in Eq. (5.16). Substituting this and the general specification of the pension policy given in Eq. (5.22) into Eq. (5.2) yields the life-cycle resources for the closed economy:

$$\begin{aligned} y_t &= w(\hat{\varepsilon}_t - \alpha\hat{\eta}_{t-1})\left[1 - \gamma\frac{1+n+\rho\hat{\eta}_{t-1}}{1+n+\hat{\eta}_{t-1}}\right] + \gamma w(\hat{\varepsilon}_{t+1} - \alpha\hat{\eta}_t)\frac{1+n+\rho\hat{\eta}_t}{1+r} \\ &= w(\hat{\varepsilon}_t - \alpha\hat{\eta}_{t-1})\left[1 - \gamma\left(\rho + \frac{(1-\rho)(1+n)}{1+n+\hat{\eta}_{t-1}}\right)\right] \\ &\quad + \gamma w(\hat{\varepsilon}_{t+1} - \alpha\hat{\eta}_t)\frac{1+n+\rho\hat{\eta}_t}{1+r} \end{aligned} \tag{5.26}$$

By substituting the quadratic approximation for $(1+n+\hat{\eta}_{t-1})^{-1}$ into the second line of Eq. (5.26), one can derive the approximate solution of the expectation of life-cycle resources in the closed economy:

$$\mathrm{E}[y_t] \approx w - \gamma w\frac{r-n}{1+r} - (1-\rho)\gamma w\frac{\sigma_\eta^2}{(1+n)^2} - \alpha\gamma w\left(\frac{1-\rho}{1+n} + \frac{\rho}{1+r}\right)\sigma_\eta^2 \tag{5.27}$$

As in Sect. 5.3, the expectation of life-cycle resources are reduced by PAYG social security ($\gamma > 0$) independent from considerations concerning dynamical efficiency or inefficiency. To verify this, note that the last term in Eq. (5.27) is unambiguously negative for $\gamma > 0$ and $\rho \in [0, 1]$. While the expectation of life-cycle resources in the closed economy is not a strictly positive function in ρ for all parameter values, as it is the case in Sect. 5.4.1, the restriction that needs to be satisfied for $\mathrm{E}[y_t]$ being maximized at $\rho = 1$ requires only that the economy is not in an extremely dynamically inefficient region ($n \gg r$).[17] So again, the expectation of life-cycle resources is maximized at $\rho = 1$ (DC) for realistic parameter values.

[17] The precise restriction is $\alpha(n-r)\frac{1+n}{1+r} < 1$.

The approximate variance can be obtained by substituting the linear approximation of $(1 + n + \hat{\eta}_{t-1})^{-1}$ into the second line of Eq. (5.26) (see Appendix A.5):

$$\begin{aligned}
&\mathrm{Var}^{la}[y_t] \\
&= w^2 \Biggl\{ \left[(1-\gamma)^2 + \gamma^2 \left(\frac{1+n}{1+r} \right)^2 \right] (\sigma_\varepsilon^2 + \alpha^2 \sigma_\eta^2) \\
&\quad + \left[(1-\rho)^2 \left(\frac{\gamma}{1+n} \right)^2 + \rho^2 \left(\frac{\gamma}{1+r} \right)^2 \right] \left[\sigma_\eta^2 (1 + \sigma_\varepsilon^2) + 2\alpha^2 \sigma_\eta^4 \right] \\
&\quad -2 \left[(1-\rho) \frac{\gamma(1-\gamma)}{1+n} + \rho(1+n) \left(\frac{\gamma}{1+r} \right)^2 \right] \alpha \sigma_\eta^2 \Biggr\}.
\end{aligned} \tag{5.28}$$

Eq. (5.28) is the general version of all other cases derived earlier. Accordingly, all other results discussed until now can be reproduced by choosing the correct parameters: $\alpha = 0$ and $\rho \in \{0, 1\}$ will generate the results of Sect. 5.2, $\alpha > 0$ and $\rho \in \{0, 1\}$ produce the variances given in Sect. 5.3 and finally $\alpha = 0$ and $\rho \in [0, 1]$ yields Eq. (5.25) discussed in Sect. 5.4.1. Not surprisingly, the interpretation of Eq. (5.28) draws on the different results derived in the previous Sections. The first line shows the variance reducing effect of PAYG social security when labor income is subject to two kinds of risks: the risk of fluctuating labor income because of uncertain productivity, and uncertain demographic growth is reduced by sharing this risk across generations (compare Sect. 5.3). The second line shows that the risk of an uncertain return form the social security scheme, which we branded the "necessary evil" of PAYG social security, can be minimized by choosing a scheme that shares the demographic shock roughly in even parts (ρ near 0.5 depending on r and n). The third line shows the "covariance effect" already discussed in Sect. 5.3: because the replacement rate and the underlying labor income that determine the transfer payment move in opposite directions, the riskiness of the return of the pension scheme is automatically reduced in the closed economy. The "covariance effect" in Eq. (5.28) is equivalent to the respective effects in the polar cases of Sect. 5.3 (see equations (5.19) and (5.20)) with ρ determining the weights of the the single covariance terms. Note that the variance increasing effect of line two is reduced in comparison to Sect. 5.3, but the variance reducing "covariance effect" in line three is not.

The risk minimizing level for demographic indexation ρ^* in the closed economy is given by:[18]

$$\rho^* = \frac{1}{1 + \left(\frac{1+n}{1+r} \right)^2} + \alpha(1+n) \frac{1 + \left(\frac{1+n}{1+r} \right)^2 - \frac{1}{\gamma}}{\left[1 + \left(\frac{1+n}{1+r} \right)^2 \right] \left[1 + \sigma_\varepsilon^2 + 2\alpha^2 \sigma_\eta^2 \right]}. \tag{5.29}$$

[18]Note that ρ^* will be the lower bound for the welfare maximizing level of ρ, since the expectation of life-cycle resources is increasing in ρ. Only for highly risk averse individuals will ρ^* also be the welfare maximizing value of ρ. For risk neutral individuals, the optimal policy will always be a defined contribution scheme.

The first term on the RHS of Eq. (5.29) is familiar from the optimal demographic indexation in the small open economy: depending on r and n, the variance can be minimized by choosing a level for the demographic indexation that roughly equals 0.5. In the closed economy, the second term in Eq. (5.29) is added. Three properties of ρ^* in the closed economy are noteworthy. First, ρ^* is an increasing function in γ. Second, ρ^* should be smaller in the closed economy than in the small open economy, if $1 + (\frac{1+n}{1+r})^2 < \frac{1}{\gamma}$. For $r = n$, this condition is reduced to $\gamma < \frac{1}{2}$. Third, for $1 + (\frac{1+n}{1+r})^2 < \frac{1}{\gamma}$, the optimal demographic indexation, ρ^*, is a decreasing function in α for realistic parameter values.[19] All three properties of ρ^* are closely linked to the difference in the "covariance effects" of DC and DB already discussed in Sect. 5.3. Because of the "covariance effect", DC provides better insurance for the $\hat{\eta}_t$ shock and DB provides better insurance for the $\hat{\eta}_{t-1}$ shock. Depending on the size of social security ($\gamma < 0.5$ for $r = n$), $\hat{\eta}_{t-1}$ has a larger influence on life-cycle resources than $\hat{\eta}_t$ and therefore DB dominates DC from a risk perspective. This is equivalent here, only that the optimal policy does not swing from one extreme to the other, but gradually adjusts towards one of the two schemes. The third property points out that for larger α, i.e. a stronger influence of demographics on the wage rate, this "covariance effect" has a stronger influence on the optimal choice of ρ.

Independent of what model serves better to describe reality for the problem at hand – small open economy or closed economy – it is noteworthy that in either case, a "mixed demographic indexation", that roughly divides the demographic shock equally between the old and the young, seems superior to the polar cases DC and DB. A similar argument has been made by Wagener (2003a) for a mix of a fixed contribution and a fixed replacement scheme in a world with deterministic demography. A general notion appears: social security is an important device for sharing risks. As such, the benefit formula should be tuned in order to guarantee optimal risk sharing. Not surprisingly, risks are shared best when they are allocated in the broadest sense possible. Choosing a pension schemes that is a mixture between the various types of imaginable systems tends to foster risk sharing.

5.5 Further Issues

5.5.1 Uncertain Life-Expectation

Until now, the demographic uncertainty was restricted to fluctuation in the population growth rate, i.e. the fertility rate. Generally, the return within a social security scheme is dependent on the old-age dependency ratio (OAD). In our highly stylized setting, this is equal to: $\text{OAD}_t \equiv \frac{N_t^o}{N_t^y}$. This ratio may change due to either an unexpected rise or decline of the young cohort or the old cohort alive in period t. An unexpected change of the size of the young cohort is due to changes in the fertility rate (see above). The size of the old generation (N_t^o) can deviate from its expectation if the

[19]The exact conditions for ρ^* decreasing in α are: $1 + (\frac{1+n}{1+r})^2 < \frac{1}{\gamma}$ and $\alpha < \sqrt{\frac{1+\sigma_\varepsilon^2}{2\sigma_\eta^2}}$.

life-expectancy is not deterministic. We will shortly present the analysis conducted above for the case with stochastic longevity on an aggregate level. It seems reasonable to assume that closed economy-effects on factor prices should not be expected from shocks to longevity.[20] Thus, we can restrict our analysis to the simple case where labor income is independent of the demographic shock. Also, we will not take the detour of looking at the polar cases. Instead we jump directly to optimal demographic indexation.[21]

The model is changed in the setup of the demographic process. Assume that the length of the second period in life is equal to its expectation p plus a stochastic component $\hat{\pi}$ that is n.i.i.d. with mean zero and variance σ_π^2.[22] The size of the old generation in period t in respect to the cumulative time length lived during retirement is given by:

$$N_t^o = (p + \hat{\pi}_t)N_{t-1}^y. \tag{5.30}$$

Inserting the equations for demographic growth and life-expectancy into the old-age dependency ratio yields:

$$\mathrm{OAD}_t = \frac{p + \hat{\pi}_t}{1 + n + \hat{\eta}_{t-1}}. \tag{5.31}$$

For the sake of a tractable presentation, we will assume that population growth is deterministic, i.e. that $\mathrm{E}[\hat{\eta}] = 0$ with probability one. The old-age dependency ratio then equals $\mathrm{OAD}_t = (p + \hat{\pi}_t)(1 + n)^{-1}$.

The demographic indexation of the benefit rule is designed such that the policy-parameter θ specifies to what degree the young generation's per-capita tax payment is determined by the expectation or by the realization of the old-age dependency ratio:

$$\tau_t = \gamma w_t \left(\frac{\theta\, \mathrm{E}[\mathrm{OAD}_t] + (1-\theta)\, \mathrm{OAD}_t}{\mathrm{E}[\mathrm{OAD}_t]} \right) = \gamma w_t \left(1 + (1-\theta)\frac{\hat{\pi}_t}{p} \right). \tag{5.32}$$

The corresponding policy for transfer payments is derived from Eq. (5.32) and the no-deficit-condition for the pension scheme:

$$\beta_t = \frac{\tau_t}{\mathrm{OAD}_t} = \gamma w_t \frac{1+n}{p} \cdot \frac{p + (1-\theta)\hat{\pi}_t}{p + \hat{\pi}_t} = \gamma w_t \frac{1+n}{p} \left((1-\theta) + \theta \frac{p}{p + \hat{\pi}_t} \right). \tag{5.33}$$

Again, the two polar cases, where either only taxes or only transfers are adjusted, are special cases of this more general policy rule. Specifically, $\theta = 1$ is equal to the DC case, and $\theta = 0$ is equal to the DB case.

[20] The issue of demand-side effects during aging has been raised elsewhere. We abstract from this possibility.

[21] In a similar line of argument, Bohn (2002) has proposed to introduce longevity indexed government bonds.

[22] For the budget of the public pension program, this is equivalently to a scenario where the retirement period is of equal length as the youth period, but only a fraction of $p + \hat{\pi}$ percent of the young generation survives period one and will reach retirement.

The life-cycle resources under this scenario are given by

$$y_t = w_t - \gamma w_t \left(1 - (1-\theta)\frac{\hat{\pi}_t}{p}\right) + \gamma w_{t+1}\frac{1+n}{1+r}\cdot\frac{1}{p}\left((1-\theta) + \theta\frac{p}{p+\pi_{t+1}}\right), \tag{5.34}$$

and the expectation of life-cycle resources equals:

$$\begin{aligned} \mathrm{E}[y_t] &= w(1-\gamma) + \gamma w\frac{1+n}{1+r}\cdot\frac{1}{p}\left((1-\theta) + \theta p\,\mathrm{E}\left[\frac{1}{p+\hat{\pi}_{t+1}}\right]\right) \\ &\approx w(1-\gamma) + \gamma w\frac{1+n}{1+r}\cdot\frac{1}{p} + \gamma w\theta\frac{1+n}{1+r}\cdot\frac{1}{p^3}\sigma_\pi^2. \end{aligned} \tag{5.35}$$

Assuming that θ is restricted to be $\theta \in [0,1]$ the expectation of life-cycle resources is again maximized in the pure defined contribution case, where $\theta = 1$.[23]

The variance of life-cycle resources can be derived as before.

$$\begin{aligned} \mathrm{Var}^{la}[y] = w^2 \Bigg\{ & \left[(1-\gamma)^2 + \gamma^2\left(\frac{1+n}{1+r}\cdot\frac{1}{p}\right)^2\right]\sigma_\varepsilon^2 \\ & + \gamma^2\left[(1-\theta)^2 + \theta^2\left(\frac{1+n}{1+r}\cdot\frac{1}{p}\right)^2\right](1+\sigma_\varepsilon^2)\frac{\sigma_\pi^2}{p^2}\Bigg\}. \end{aligned} \tag{5.36}$$

The variance exhibits a striking similarity to the variance under optimal demographic indexation in the small open economy case for stochastic fertility. In fact, they are identical if $p = 1$. The discussion of the results derived in Sect. 5.4.1 can thus be directly transferred to the case with stochastic longevity. The interpretation of insuring longevity-risks on a generational level may actually be more appealing than the insurance of fertility risks. Complications arising in connection with stochastic fertility such as factor-income effects, long-term predictability of labor force growth, and the issue of endogenous fertility decisions do need not be considered for the case of stochastic longevity.

5.5.2 *Ex-Ante* versus *Ex-Post* Risk Sharing

Rangel and Zeckhauser (2001), Wagener (2003b), and Matsen and Thøgersen (2004) differentiate between two types of risk sharing: traditional risk sharing and Rawlsian risk sharing. The former type of risk sharing is similar to the earlier mentioned *ex-post* risk sharing, where the realization of period t variables is taken as given. The latter type is similar to the concept of ex-ante risk sharing that was used here. According to the latter concept, people still face a random realization of variables affecting the young period of their lives. This concept may be illustrated by imagining all individuals of the different generations are present behind a "veil of ignorance",

[23] To verify this, note that for $\hat{\pi}_{t+1} > -p$, the function $\frac{1}{p+\hat{\pi}_{t+1}}$ is concave in the random variable and therefore from Jensen's inequality, $p\,\mathrm{E}[(p+\hat{\pi}_{t+1})^{-1}] > 1$.

where the "veil of ignorance" symbolizes the uncertainty which type of generation ("lucky" or "unlucky") the single individual will be born into. In the case of Rawlsian risk sharing, Matsen and Thøgersen (2004) eliminate stochastic population growth.[24]

In general, risk sharing is more effective under the ex-ante perspective. Also, strategies that are optimal from the ex-ante perspective may not be time-consistent (cf. Rangel and Zeckhauser (2001)). Not surprisingly, the preference towards an insurance contract changes considerably once the uncertainty is resolved. More specifically, Wagener (2003b) shows that the result of Thøgersen (1998) cited above is reversed for *ex-post* risk sharing.

The role of land as a durable and fixed productive input for the efficiency of intertemporal allocation has been alluded to in Sect. 2.3.3. For the case with uncertainty, Demange (2002) proves that the existence of land and sequentially complete markets will guarantee the optimality of a pure market equilibrium. If land exists and markets are not sequentially complete, Demange (2002) shows that a "constrained" optimum[25] only prevails for *interim* (or *ex-post*) optimality.[26] This result has also been presented specifically for the case with demographic shocks by Demange and Laroque (1999).[27] However, social security may be Pareto improving in such a constrained setting if an *ex-ante* optimality condition is being used. Therefore, we have concentrated on this type of risk sharing in our analysis.

5.6 Summary

Expanding the analysis of Thøgersen (1998) by an uncertain population growth rate may put the intergenerational risk sharing features of wage-indexed pension programs into perspective. In a small open economy, the reduction of labor income risk via these programs is bought at the expense of an added demographic risk that is not present in a fully funded system.

The inclusion of the stochastic population growth rate makes it necessary to specify how the realization of the demographic random variable will affect contributions and benefit payments of the pension program. In Sections 5.2 and 5.3, the policy options are restricted to the two extreme cases: defined contribution wage indexation and defined benefit wage indexation. The essential difference between the two schemes is the exposure to the demographic shock. Under defined contribution, the risk of the demographic shock is borne by the old generation, whereas under defined benefit, the young generation's contribution payments are adjusted in response to the realization of the population growth rate. In Sect. 5.4, a more general policy concerning the demographic indexation of PAYG social security is introduced: the impact

[24]Hence, the question addressed in this Chapter is not covered by these authors.

[25]The term "constrained" is used because the incomplete marker structure is taken into account.

[26]A similar result was previously derived by Richter (1993).

[27]The results are derived for stationary long run allocations. Barbie et al. (2000) conduct a similar analysis that does not require stationary allocation.

of the population growth rate on the pension program can be split in any proportion between the currently living young and old generation.

For the two polar cases, two general distinctions are shown: first, the expectation of life-cycle resources is always greater under DC than under DB. The reason for this is that life-cycle resources are a linear function of the demographic random variable under the former, but a concave function of that variable under the latter. Second, due to the different timing of the transmission of the demographic shock within the two schemes, the discounting mechanism for the component that is associated with the variance of population growth is different: while under DC it is discounted by the gross interest rate, it is "discounted" by the gross population growth rate under DB. These two differences between DC and DB are not dependent on whether the economy is closed or open but will hold in general.

A comparison of DC versus DB in a small open economy (Sect. 5.2) has shown that if the economy is dynamically efficient, DC tends to share risks better than DB. This is due to the different discounting mechanisms of the demographic shock within the two schemes. Also, the expectation of life-cycle resources is larger under DC than under DB. This leads to the conclusion that in a small open economy from an ex-ante perspective a defined contribution scheme is to be preferred to a defined benefit scheme for $r > n$ (dynamic efficiency).

In Sect. 5.3, we allow for a macroeconomic feedback effect of population growth on labor income in order to mimic general equilibrium effects of a closed economy. The results between Sect. 5.2 and Sect. 5.3 differ significantly. First, in a small open economy life-cycle resources are not subject to demographic risks without a PAYG pension program. Gains from sharing labor income risk between generations via social security are therefore reduced because a demographic risk must be added. In a closed economy, this is very different: labor income is subject to demographic changes itself and the risk of uncertain labor income due to fertility shocks is also reduced via social security. However, social security still adds uncertainty to life-cycle resources because of the uncertain return from the pension scheme. Thus, depending on the size of the macroeconomic feedback effect, demographic risks are actually reduced by social security in the closed economy instead of increased, as it is the case in the small open economy. However, this insurance against the influence of demographic shocks on labor income is not for free: under both policies, the expectation of life-cycle resources is reduced because of the demographic uncertainty. This reduction is independent of dynamic efficiency or dynamic inefficiency and therefore comes in addition to the well known results concerning the return of PAYG social security; see Aaron (1966).

The second distinction between the small open economy and the closed economy concerns the specific design of wage-indexed social security: in the closed economy life-cycle resources are touched by the realizations of the two demographic shocks, $\hat{\eta}_{t-1}$ and $\hat{\eta}_t$, in three ways. First, $\hat{\eta}_{t-1}$ affects generation t's own labor income. Second, $\hat{\eta}_t$ affects the labor income of generation $t+1$, which, for a wage-indexed pension scheme, affects the retirement payments of generation t. And thirdly, depending on the policy, either the replacement rate is adjusted in response to the shock $\hat{\eta}_t$ (DC) or the contribution rate is adjusted in response to $\hat{\eta}_{t-1}$ (DB). The respective third effect will always be in a different direction than the first (DB) or the second effect (DC). The combination of the third and first effect or third and second effect, respectively, can then be interpreted as a "covariance" between the respective basis and the applicable contribution rate (DB) or replacement rate (DC). Since this "covariance" is negative, it provides additional insurance against movements in labor income due to demographic changes. The "covariance" differs between DC and DB with respect to which shock is covered: DB provides this additional insurance for the shock $\hat{\eta}_{t-1}$, while DC does so for n_t. Hence, if a generation's own labor income has the largest weight in that generation's life-cycle resources, it can generally be assumed that DB provides better insurance than DC.

In Sect. 5.4, a more general policy specification concerning the demographic indexation of the pension program is introduced: the influence of the demographic shock of a single period on the payments to and from the social security scheme can be split in any given proportion. Under this general specification, the risk that is added to life-cycle resources by PAYG social security because of the uncertain return on the contributions paid into the scheme can be significantly reduced by choosing the correct demographic indexation. Specifically, a policy that splits the financial effect of the demographic random variable on social security roughly equally between the living generations is suited best to reduce the variance of life-cycle resources. Also, the extreme difference between the small open economy and the closed economy concerning the optimal choice of DC and DB is no longer present. Depending on various parameters the optimal demographic indexation only gradually moves away from the "split-evenly" rule. Finally, we have shown that the optimal demographic indexation can also be interpreted in terms of stochastic longevity.

Chapter 6
Demographics and Political Risks of Benefit Rule Changes: A Case Study for Germany

In this chapter, we look at the risk of legal changes to retirement benefits from the public pension program in Germany due to changes in the specification of the benefit formula.[1] Specifically, we address two issues here: first, we calculate the "political riskiness" of the German public pension scheme. Second, we estimate how demographic developments have triggered the changes to the benefit formula.

By "political riskiness", we mean the risk that the generosity of a pay-as-you-go pension scheme varies due to legislative changes of the benefit formula. These alterations to the benefit rule obviously constitute a source of risk for the individuals life-cycle resources. McHale (2001) has pointed out this source of risk for old-age income in a conference volume that actually focused on risk aspects of investment based social security reforms edited by Campbell and Feldstein (2001). In order to quantify this political risk, we construct a measure of the relative generosity of the benefit formula and track this measure for single cohorts over the time-span from 1970 to 2001. This measure is limited, since it is constructed around the standard retiree (*Eckrentner*) who has paid 45 years of contributions on a gross salary equal to the average income of every year. While this constitutes a very unrealistic time-path of contribution payments, this is not such a serious restriction on our measure, since the time path is not important in calculating the relative generosity of the benefit rule.

Additionally, we consider two alternative scenarios. In scenario two, which will be our benchmark, the retiree will always make use of the possibility to retire at the earliest date possible for the regular pension payment. The third scenario adds the assumption that the retiree has spent seven years in higher education and is making use of the early-retirement option. This will be the scenario with the highest volatility in relative generosity. Our measure is based on using the benefit formula as it is effective in a given year and assuming that it will not be amended from then on. Applying this principle, we calculate the gross social security wealth (SSW) in present value at the time of entering retirement for each year from 1970 until 2001. The relative generosity of the pension scheme is then defined as the fraction of SSW_t in each respective year t divided by the social security wealth that would have resulted if the laws concerning the benefit rule from 1970 were still effective. This measure of relative generosity will then show how the generosity of the German public pension scheme has increased or decreased over time. We apply this procedure to construct a time series of relative

[1]This Chapter draws extensively on previous work by Borgmann and Heidler (2003).

generosity for (a) single cohorts over time (RGC_{birth}) and (b) for individuals who are aged 45 and 62 in each year (RGT_{age}).[2]

Our findings for the cohort born in 1950 are that – for the benchmark-case of early retirement – the relative generosity of the benefit formula, which is defined to be unity in 1970, will increase to 1.13 in 1972 and gradually decreases to 0.63 in 2001. In the scenario with seven years of higher education this deterioration of relative generosity amounts to 0.49, indicating that the generosity has been cut by 58 percent when comparing the current level of generosity to the zenith in 1972. For the cohorts born in 1940 and 1930, the losses in this third scenario between the highest level of generosity in 1972 and the lowest level in 2001 amount to approximately 53 and 42 percent, respectively.

The second aim of this chapter is to provide an analysis on what drives the changes in the benefit formula. Specifically, we want to estimate a policy reaction function for the relative generosity of the German pension scheme. The main focus when estimating this function is how demographic developments trigger changes to the benefit formula. Similar questions have been posed by McHale (2001) and Razin et al. (2002). McHale (2001) comes to the conclusion that a relative increase in the old-age dependency ratio leads to a more than one-to-one relative increase in the share of old-age cash benefits to GDP. Razin et al. (2002) analyze the size of the entire welfare state and the labor tax rate and find that both the tax rate and the generosity of the transfer program measured in transfers per capita decrease with the dependency ratio.

Our study is more in the line of McHale, since we concentrate only on the payments from the mandatory pension scheme (*Gesetzliche Rentenversicherung*). An important deviation from both other studies is that we are not estimating an actual time-series of transfer payments, but rather a measure of relative generosity (RGT_{age}) that we construct from the benefit rule as legislated during the time from 1970 until 2001. This allows us to include reforms that are only gradually phased in and therefore are not included in actual data yet or are not visible in the data at the time the reform is made. Also, we do not only regress on current demographic changes, but we also test the influence of future demographic changes.

Our results suggest that not only the current but also the future level of the old-age dependency ratio has a significant influence on the current level of generosity. An increase of the current old-age dependency ratio tends to raise the generosity, while increases of the future old-age dependency ratio reduce generosity. Surprisingly, the elasticity of the generosity to demographic changes is larger in absolute terms for the future old-age dependency ratio than that elasticity for the current old-age dependency ratio. Furthermore, the result of a negative elasticity of the generosity measure with respect to the future old-age dependency ratio holds for both the relative measure of generosity for the 45-year old (RGT_{45}) and the 62-year old (RGT_{62}). This contradicts the result of McHale (2001) that persons just entering retirement are

[2] For most of our analysis, we keep life-expectation constant over time for each respective cohort. In Fig. 6.6b, we also report our measure of generosity when changes in life-expectation are taken into account.

protected from reductions in generosity. The contribution rate, on the other hand, is much less influenced by demographics than the relative generosity.

Finally, to put past changes into context of what might still be ahead of us, we apply the method of generational accounting to assess the sustainability of the social security system under the current valid legislation. We propose a new indicator for the measurement of sustainability that calculates the size of necessary changes when reforms are only possible in a piecemeal fashion. According to our results, reductions from the 2001-level of benefits will have to be in the range from 30 to over 40 percent under this “soft transition” scenario. Instead, if reductions were fully implemented straight away, the necessary cuts would amount to only roughly 20 percent.

The chapter is organized as follows. Section 6.1 summarizes and categorizes all changes to the benefit rule of the German public pension scheme since 1957. In Sect. 6.2, we quantify these changes in terms of generosity of the pension system for specific cohorts and scenarios. The current demographic situation and past population projections are summarized in Sect. 6.3. In Sect. 6.4, we look at demographic developments and changes of the benefit rule. Specifically, we modify our indicator of relative generosity such that we are able to measure changes as a time series in Sect. 6.4.1. In Sect. 6.4.2, we use this data to estimate a policy function of changes to the generosity of the public pension scheme. To conclude this chapter, we turn the perspective from the past to the future and we quantify the sustainability of social security under the current status quo in Sect. 6.5.

6.1 A Chronicle of the German Benefit Formula

6.1.1 The History of the German *Gesetzliche Rentenversicherung*

Germany has been the first country in the world to set up a public pension scheme. The first steps towards such a scheme were taken in 1889, when insurance against disability was introduced for workers. The system rapidly expanded towards a regular pension scheme.[3] However, it wasn’t until 1957 that the benefits from the pension scheme were indexed to the growth rate of labor income, and it took another twelve years until the pension scheme became a purely pay-as-you-go financed social security system.[4] These two features, pay-as-you-go financing and wage-indexed earnings related benefits, are the essential characteristics of what is often called a “Bismarckian” system of public pension provision that is predominant in continental Europe.

[3] For a chronicle of the German public pension scheme from 1889 to 1957, see Köhler (1990). The history of the first century of the public pension in Germany is extensively covered in the Festschrift edited by Ruland (1990) and in Kohl (1990). Helpful overviews of the German public pension system can be found in VDR (2002a), Steffen (2002a,b), and Börsch-Supan (1999).

[4] Next to the introduction of the benefit formula, the reform in 1957 also marked the begin of a unified pension scheme (*GRV*) under which the same laws apply to the pension systems of the workers (*ArV*) and the employees (*AnV*). The miners pension scheme (*KnV*) was integrated somewhat later. Today, about 90 percent of all employed workers are covered by the *GRV*.

The pension system is financed via flat rate contributions on wage income until a certain income threshold is reached (*Beitragsbemessungsgrenze*). The contribution rate has varied between 14 percent (1957–1967) and 20.3 percent (1997–1999) and is currently at 19.5 percent. Next to the payroll contributions, the state subsidizes the public pension scheme at a level that has been in the range between one fifth and one third of total expenditures. Recent legislation – the so called "green-tax-reform" – will most likely stabilize this level at about one third of total expenditures. This rather large tax-financed subsidy is commonly justified by the coverage of persons and/or pension entitlement that should and would not be covered in an actuarially fair insurance.

Benefit payments are categorized into three types of pensions: pensions due to age, pensions due to incapability to work, and pensions for dependent survivors. In each of these types of pensions there are still a number of different subclasses. We will cover some of those in the following subsections, and we give an overview of theses types in Table B.2 in the Appendix. Major reforms of the benefit formula have taken place in the years 1957, 1972, 1989 (effective 1992), 1997 (planned to be effective in 1999) and 2001.

6.1.2 The Benefit Formula: A General Description

In the remaining part of this chapter, we focus only on the public pension system, the *GRV – Gesetzliche Rentenversicherung*, post 1957. Even more specifically, we concentrate on expenditures only, i.e. the benefit formula that is used to calculate the individual pension payments for the participants of the social security system. Here, we first describe the general structure of the benefit formula and we will then present a more detailed chronicle of changes to the benefit formula since 1957 in Sect. 6.1.3.

Individual pension payments are calculated with the help of the benefit formula, which is basically comprised of three components. The three components are: i) the individual eligibility calculated by the years of contribution payments weighted by personal earnings relative to the average earnings in each respective year, ii) an adjustment factor that is dependent on the type of pension and possible deductions for early retirement, and iii) the indexation of the benefits. The first two of these components are dependent on an individuals employment history and choice on when to retire. The personal eligibility is also increased for years spent in education and child rearing.

Since the wage-indexed pension has been introduced in 1957, the general structure of the benefit formula has only been changed once under the so-called "Social Security Reform Act of 1992" (RRG92).[5] The formulas pre and post 1992 respectively are shown in Table 6.1.[6] As can be seen in row one of this table, both formulas fit into the classification of the three components given above.

[5]The rules on the single components have however changed numerous times. Specifically the indexation of the benefits has undergone three fundamental reforms and several minor changes. Compare Sect. 6.1.3, specifically Table 6.3.

[6]The representation of the benefit formulas and their components in Tables 6.1 and 6.3 is based on Ruland (1989) and SVR (2001,2002).

Table 6.1. Benefit formula for periods 1957–1992 and 1992–today (numerical values are for the standard retiree)

year	**individual eligibility**	x			**adjustment factors** (depend. on retirement age and pension-type)	x	**index-ation**	=	**benefits**	
1957 –1992	Vj x $pVhs$	x			St	x	AB	=	annual benefits	monthly benefits
1957	45 x 100%	x			1.5%	x	4281	=	2,890	241 DM
1992	45 x 100%	x			1.5%	x	33149	=	22,376	1,865 DM
RRG92	↓		**new** ↓		$1.5\% \cdot 33149/12 = 41.44$ transformation ↘		↘			
1992 -today	EP	x	ZF	x	RF	x	AR	=		monthly benefits
1992	45	x	1	x	1	x	41.44	=		1,865 DM
2002	45	x	1	x	1	x	25.86	=		1,164 €

with

Vj	years of coverage	Versicherungsjahre
$pVhs$	personal earnings relative to average earnings	pers. Vomhundertsatz
St	scaling of benefits depending on type of pension	Steigerungssatz
AB	statistical measure of average earnings of employed during past years	allgemeine Bemessungsgrundlage
EP	years of coverage scaled by average earnings	Entgeltpunkte
ZF	adjustment factor for early retirement	Zugangsfaktor
RF	adjustment factor depending on type of pension	Rentenartfaktor
AR	statistical measure of value of entitlements	aktueller Rentenwert

There are three differences between the two formulas; nonetheless the structure remains the same. Firstly, the components Vj and $pVhs$ that relate years of coverage to relative earnings during time of service have been joint together into one variable EP. This step has only minor consequences since it mainly changes how credits for non-working years that nevertheless increase eligibility are valued (especially years spent in education). Secondly, an adjustment factor ZF is introduced in 1989 (but will only be applied after 2000) that reduces or increases benefits depending on the choice of the retirement age; cf. Sect. 6.1.4. And thirdly, the combination of $St \cdot AB$ is being numerically transformed into values for RF and AR from a calculation that is based on annual values into monthly values. In 1992, where both formulas yield the identical benefit payments, the AR_{1992} was defined by $St \cdot AB_{1992}/12$ (see Table 6.1). The calculation of AR after 1992 is, however, fundamentally different from the calculation of AB until 1992 (see Table 6.3). At the same time the factor St was numerically transformed into a new factor RF. The respective values for the standard retiree are 1,5 percent for St until 1992 and unity for RF from 1992 onwards.[7] The relative difference between the different types of pensions remains the same for St and RF. Therefore, the two factors serve exactly the same purpose.

In Table 6.1, we also show the numerical calculation of benefits for selective years. The numerical values represent the so called standard retiree (*Eckrentner*) who has 45 years of service with the further assumption that the standard retiree has always earned exactly the average income in every one of those 45 years during his working life. Also, it is assumed that the standard retiree enters retirement at the regular retirement age for pensions due to age at the age of 65. He[8] will therefore receive a pension due to age without any reductions or increases. The standard retiree will be the point of departure for the scenarios chosen in Sections 6.2 and 6.4.

6.1.3 The Benefit Formula from 1957 to 2001: A "Moving" History

As described earlier, 1957 marked the beginning of the wage-indexation of the German public pension scheme. A dynamic component was introduced by adjusting pension payments annually to the mean growth rate of the gross average labor income of the past years.

Since then, the benefit formula or some of its components have undergone major changes in 1972 (early retirement), 1989 (net indexation, effective 1992), 1997 (demographic factor, effective 1999), and 2001 (modified gross indexation). We will first describe the major reforms chronologically and then touch on some further changes to specific categories of the pension formula. All changes are summarized in Table 6.2. Furthermore, it is indicated whether the changes have increased (+) or decreased (−) the generosity of the benefit formula. The table is chronologically[9] organized according to the three major components described above: eligibility, adjustment factors[10], and indexation. Additionally, we highlight which of these changes will be relevant for our three scenarios used for the analysis of relative generosity later.

[7]The values of the new adjustment factor (RF) for different types of pensions are given in Table B.2 in the Appendix.

[8]The usage of the male gender is intentionally and necessary because other rules apply for females.

[9]Because in Sections 6.2 and 6.4 we are interested in the generosity of the benefit rule under the effective law at the different points in time, the chronological order depends on the time of legislation and not on the time of implementation.

[10]This column also includes further requirements for personal and general eligibility of the pension due to incapability to work and the survivor benefits.

Table 6.2. Changes to the generosity of the German benefit formula

time of legis-lation	time of implemen-tation of changes	name of law	individual eligibility			type of pension / adjustment factors				indexation of pension			others:
			working years	credits for other activities		normal retirement	other pensions			date of adjust-ment	statistical calc. of adjustment	fundamental changes	health ins. contr.
				education	child-rearing		regular early retirement	incapability to work	survivors benefits				
			1	2	3	4	5	6	7	8	9	10	11
1957	**1957-**	**Renten-reform**	**Major reform of the public pension sxstem: Introduction of gross wage indexation (partially funded until 1968)**										
1972	**1972-**	**Renten-reform**	+ (MinE)*				+	+		+			
1977	1977-	20. RAG		-				-		-	-		
1978	1979-1981 1980-	21. RAG RVÄndG						+				- 4.5% and 4%	
1982	1983-1986	HBeglG								-	-		-
1983	1984-	HBeglG		-		+ (15->5)*		-	-		-		
1985	1986-	HEZG			+				+				
1989	**1992-2019**	**RRG '92**	- (MinE)*	-	+		-					- **net index**	
1996	1996-2001	WFG BeutrEntlG		-				-					
1997	**1999-2011**	**RRG '99**			+		+	-				- **demogr. comp.**	
1998	1999							+				+ non-applic. of demogr. comp.	
1999	2000	HSanG										- Inflation index.	
2000	2001							-					
2001	**2001-2011**	**AVmEG**			+				-			- **mod. gross index**	
Relevant for scenarios			**1,2,3**	**3**	**none**	**1,2,3**	**2.3**	**none**		**1,2,3**			**1,2,3**

In 1972, the generosity of the pensions was extensively increased by allowing early retirement for male individuals after reaching the age of 63 years if they have at least 35 years of coverage. This early retirement option was introduced without a compensating reduction of transfer payments.[11] The official term of the early retirement option is "pensions due to age for long-term contribution-payers" (*Altersrente für langjährige Versicherte*), which we will coin as "regular early retirement". Along with this regular early retirement option, a wide variety of further paths into early retirement were introduced. In our analysis of the relative generosity, we will nevertheless only consider the regular early retirement option. For completeness, Sect. 6.1.4 covers the different paths into retirement. Finally, the 1972 reform increased the generosity of the public pension system for low-income earners with the introduction of a minimum credit for one year of contribution payments. Individuals with an income that was below 75 percent of the average earnings were accredited an entitlement as if they had paid contributions on the threshold value of 75 percent (*Rente nach Mindesteinkommen*: MinE).

After the expansion of the generosity of the benefit formula with the reform of 1972, the generosity has steadily been reduced in the following decades. The only exemption from this general reduction of generosity is the introduction and expansion of benefit entitlements for rearing children.[12]

The next truly fundamental reform was legislated in 1989 and effective from 1992 onwards.[13] This reform changed the general form of the benefit formula (see Table 6.1), replaced the gross earnings indexation by a net earnings indexation (cf. Table 6.3), increased the minimum age (with a long transition period) for a number of paths into retirement (cf. Sect. 6.1.4), and introduced an adjustment factor for early retirement (ZF). In addition, the minimum earnings rule was abrogated and coverage for years spent in education severely reduced. The only enhancement of generosity of the RRG92 was an increase of entitlement for child-rearing (three years instead of one). As one can see, the RRG92 was the most comprehensive reform since the introduction of the indexed benefit formula. Even today, the long-term effects of this reform on the financial situation of the public pension scheme can hardly be captured in full detail.

In 1997, under the pension reform act of 1999 (RRG99), some paths into retirement were abolished and the net-wage indexation was appended by a demographic factor (cf. Table 6.3). This demographic factor was supposed to take changes to

[11] The pension payments were of course reduced because the retirees had two years less of contribution payments at retirement in comparison to the pre 1972 retiree, who worked (more or less) until 65 (see Sect. 6.1.4).

[12] Since the beginning of the 1980s politicians and scientists alike have stressed that the rules of the pension scheme do not do justice to the true nature of a pay-as-you-go financed pension scheme that is in fact a three-way-generational contract, in which the working generation is supporting both the old and the young, instead of just a two-way contract between the working population and the retirees. See e.g. Borchert (1981), Eekhoff (1985) and Sinn (1998).

[13] Some measures of this reform will not be effective before 2018, thirty years after legislation.

longevity into account, but it was suspended before it came to application and was later replaced by an entirely new indexation of the benefits.

The pension reform of 2001 was, for the time being, the last reform of the German benefit formula. With this reform, the indexation of benefits has again been changed fundamentally. The former net indexation (including the suspended demographic factor) is now replaced by a modified gross indexation. The dynamic factor AR is now determined by the growth of gross earnings and by changes in the deductions for the provision of old-age income. These deductions are defined much broader than before because they include employees' and employers' contributions to the *GRV* and in addition, a fixed percentage to a facultative individual account for retirement benefits, the so-called *Riester-Pension*.[14]

Turning to the changes of specific parts of the benefit formula, we concentrate on those that will be of relevance for the scenarios based on the standard retiree that will be the center of interest in the proceeding Sections.

Education. During 1957 and 1989, a maximum of 13 years of time spent for educational purposes after the age of 16 years were accredited to personal eligibility (column 2 in Table 6.2). In 1989, the maximum number of years were reduced to seven and in 1996, another reform reduced the maximum possible years to three.[15] Next to the reduction of the upper limit for the time spent in education, the valuation of educational years was also continuously decreased.[16] Finally, the age after which educational credits can be obtained was increased from 16 to 17 years of age in 1996.

Early Retirement. The option for regular early retirement (column 5) was introduced in 1972. The pension reform act of 1992 has severely deteriorated the conditions under which future generations could choose this option. Retiring one year earlier than the regular retirement age after the year 2000 reduces pensions by 3.6 percent.[17] In 1997, the minimum age for early retirement was reduced to 62 years. We evaluate this as an increase of generosity since the deduction of 3.6 percent is still not actuarially fair (cf. Sect. 6.1.4).

[14]Note that we do not include the tax subsidies of the *Riester-Pension* when calculating our measure of generosity in Sections 6.2 and 6.4. There are two reasons for this: first, the participation is not mandatory and second, strictly speaking, this tax subsidy is not part of the public pension program, i.e. the *Sozialgesetzbuch IV*. Also, if we included this subsidy of private savings program, we would also have to consider other savings subsidies and tax breaks for employer's pension programs.

[15]The reductions of maximum eligible years of the RRG92 and in 1996 were only applied after a transition period until 2004 and 2001, respectively.

[16]The educational credits were evaluated at the personal average earnings ($pVhs$) until 1965. From 1966 until 1977 lump-sum credits were given depending on the kind of education. A year of university education was evaluated at approximately 200 percent p.a. From 1977 until 1983 all types of education were evaluated at 100 percent p.a. From 1983 onwards years spent in education after 1965 were evaluated at 90 percent. The RRG92 reduced the lump-sum credit to 0.75 EP.

[17]During the transition period between 2001 and 2006 the reductions are lower.

Indexation. The indexation of benefits has been changed numerous times during the past four decades. The fundamental changes have already been covered above. There are however other, on first sight minor, modifications that can have an influence on the generosity of the pension scheme. These can be classified into changes to the date of adjustment (column 8) and the way past average earnings are used to calculate the indexation of benefits (column 9). In 1972, the adjustment-date was moved from the 1st of January to the 1st of July. Whereas in 1978/79, the adjustment-date was moved back to the 1st of January and in 1983 again to the 1st of July, where it has stayed since (for the Old Laender). Changing the timing of adjustment increased the generosity of the social security program in 1972 because pensions were increased after six months instead of after twelve. The other two times the new date of adjustment reduced the generosity because pensions were not adjusted for eighteen months. The formula to calculate the indexation from past average earnings has been changed numerous times (cf. Table 6.3). For example, in 1983, the past earnings with a lag of up to five periods were used to calculate the dynamic component AB. One year later in 1984, the lag was reduced to two periods. Because the growth rate fluctuates quite significantly between years, changing the years that are considered in the formula at a given point in time can have quite an influence. Such changes have occurred three times and have always led to a reduction of the adjustments of benefits in the respective year of change. An unique measure was taken in 1978, when instead of applying an adjustment formula that is conditional on economic development, the future increases of benefit payments were fixed to 4.5 percent in 1979 and 4 percent in both 1980 and 1981. Furthermore, the pensions for existing and new retirees are no longer being differentiated by a lagged pension adjustment for existing retirees. Because the calculation of pensions for new pensioners was aligned to the adjustment of transfers to existing pensioners, this led to the only ever true reduction in benefit payments.[18]

Contributions to Health Insurance. Finally, there has been one change to the net-value of benefit payments that does not fit into any of the other categories. Starting from the 1st of July 1983, the retirees were obliged to pay contributions to the mandatory health insurance (column 11). The contribution rate on gross pensions were continuously increased until 1987. Since that time, the retirees are paying half of the total contributions rate and are therefore treated in the same manner as employees are.

6.1.4 Disability, Incapability, Unemployment and Part-Time Work: One Thousand and One Ways to Retire Even Earlier than Early

The regular retirement age of the GRV in Germany has always been and still is 65 years. Nevertheless, the average retirement age has fluctuated within the interval between 58 and 62 years of age with the current value slightly underneath (above) 60 for men (for women). The reason for this is that there have always been different ways to enter retirement other than the pensions due to age, which is the "normal"

[18] The legislator tried to hide this fact, by designing a special time path of benefits for persons entering retirement in 1978.

Table 6.3. The dynamic component of the benefit formula 1957–today

Year of legis-lation	Name of law / reform	Date of first adjustment:	Dynamic components of benefit formula: aB, aR, RAS
1957–1992	Annual benefits = Vj · pVhs · St · AB		
1957	Rentenreform 1957	01.01.1959	$AB_t = \frac{BE_{t-2}+BE_{t-3}+BE_{t-4}}{3}$ $RAS_t = \left[\left[\frac{AB_{t-1}}{AB_{t-2}}\right] - 1\right]$
1972	RRG 1972 15. RAG	01.07.1972 early adj.	$RAS_t = \left[\left[\frac{AB_t}{AB_{t-1}}\right] - 1\right]$
1977	20. RAG	01.01.1979 postponed adj. for old reitrees	$AB_t = AB_{t-1} \cdot \frac{BE_{t-1}+BE_{t-2}+BE_{t-3}}{BE_{t-2}+BE_{t-3}+BE_{t-4}}$
1978	21. RAG	01.01.1979	Equal adjustment of pensions of new and old pensioners. Therefore actual reduction of pensions for new retirees. $RAS_{1979} = 4,5\%$, $RAS_{1980} = 4\%$, $RAS_{1981} = 4\%$
1982	Rentenanpassungs- und Haushaltsbegleitgesetz 1983	01.07.1983 postponed adj.	$AB_t = AB_{t-1} \cdot \frac{BE_{t-2}+BE_{t-3}+BE_{t-4}}{BE_{t-3}+BE_{t-4}+BE_{t-5}}$
1983	Rentenanpassungs- und Haushaltsbegleitgesetz 1984	01.07.1984	$AB_t = AB_{t-1} \cdot \frac{BE_{t-1}}{BE_{t-2}}$
1992–today	Monthly benefits = EP · ZF · RF · AR		
1989	RRG 1992	01.07.1992	$AR_t = AR_{t-1} \cdot \frac{BE_{t-1}}{BE_{t-2}} \frac{NQ_{t-1}}{NQ_{t-2}} \frac{RQ_{t-2}}{RQ_{t-1}}$
1997	RRG 1999	01.07.1999	$AR_t = AR_{t-1} \cdot \frac{BE_{t-1}}{BE_{t-2}} \frac{NQ_{t-1}}{NQ_{t-2}} \frac{RQ_{t-2}}{RQ_{t-1}} \cdot DF_t$ $DF_t = 1 + (\frac{L^{65}_{t-9}}{L^{65}_{t-8}} - 1) \cdot 0.5$
1998	Gesetz zu Korrekturen in der Sozialversicherung und zur Sicherung der AN-Rechte	01.07.1999	Suspension of the demographic factor DF (for 1999 and 2000)
1999	Haushaltssanierungsgesetz	01.07.2000	$AR_t = AR_{t-1} \cdot \text{Inflation}_{t-1}$ (for 2000 and initially planned for 2001)
2001	Altersvermögens-ergänzungsgesetz	01.07.2001 01.07.2011	$AR_t = AR_{t-1} \cdot \frac{BE_{t-1}}{BE_{t-2}} \frac{100\% - AVA_{t-1} - RVB_{t-1}}{100\% - AVA_{t-2} - RVB_{t-2}}$ $AR_t = AR_{t-1} \cdot \frac{BE_{t-1}}{BE_{t-2}} \frac{90\% - AVA_{2009} - RVB_{t-1}}{90\% - AVA_{2009} - RVB_{t-2}}$

with

Vj, $pVhs$, St, AB, EP, ZF, RF, AR as in Table 6.1, and

BE	average gross earnings of employees and workers	Bruttoentgelte
RAS	adjustment to benefits that are paid to retirees who are already retired	Rentenanpassungssatz
NQ	quota of net–to-gross-earnings (including employees share of contributions to all mandatory insurances)	Nettoquote
RQ	quota of net-to-gross-pensions	Rentennettoquote
DF	demographic factor	Demographischer Faktor
L^{65}_t	remaining life-expectancy of 65 year old in year t	
AVA	Share of gross earnings for savings in individual accounts (4% in 2009)	Altersvorsorgeanteil
RVB	contributions to GRV (employers and employees share)	Beitragssatz GRV (AN+AG)

retirement type. While the title of this section overstates the possibilities to enter retirement, there are currently seven and a half[19] different ways to enter retirement without even considering survivors benefits[20]. These different paths into retirement, with the respective earliest possible age of eligibility and the necessary conditions that have to be met, are displayed in Table 6.4. A detailed discussion of these possibilities can be found in Arnds and Bonin (2002). Four of these seven paths have always existed. These are the regular pension, pension due to incapacity to work, pensions due to unemployment, and the benefits due to age for women. What we call the "regular early retirement", the pension due to age for long-term contribution-payers, has been introduced in 1972 along with the pension for severely handicapped persons. The possibility to enter retirement after part-time work has only existed since the early nineties.

These numerous and generous paths into retirement have, however, been severely cut down during the course of the last decade. The pensions due to age after unemployment and part time work and the reduced retirement age for women have been abolished. They will not be available anymore for cohorts entering retirement in the near to far future. However, there are long transition periods under the "protection of confidence". While the remaining paths into pension will offer more flexibility to retire early, i.e. the regular early retirement can be chosen at the age of 62 after 2012, this flexibility comes at the cost of reductions in benefits. The above mentioned ZF adjusts benefit payments by -3.6 (+6.0) percent per annum for each year of retiring earlier (later) than the regular retirement age.[21]

Note that retiring early, e.g. at the age of 63 instead of 65, has three consequences on the financial transactions between the individual and the pension system (cf. Sect. 2.4.2). However, during the time span of 1973 until 2000, only the effects that the retiree was not paying contributions for these two years was considered in the benefit rule of the German public pension scheme. With the legislation of the RRG92 from 1989, the other effects (receiving transfers earlier and for a longer period) will be partially taken into account in the benefit rule from the year 2001 onwards. After 2006, benefits are reduced by 0.3 percent per month (or 3.6 percent p.a.) if pensions are paid before the 65th birthday. The effect of not earning further entitlements reduces benefits by roughly 2 to 2.5 percent p.a.[22] Retiring one year early therefore reduces the monthly pension by approximately six percent in comparison to the pension that one would obtain if one had retired at age 65. According to Börsch-Supan (1999), the non-distorting reduction equals about eight percent.

[19] We speak of seven and a half, since the pension due to incapacity to work actually has to be split up in at least two cases: full incapacity and half incapacity to work, or former BU and EU pensions.

[20] There are another five types of survivors benefits (cf. Table B.2, where we have not included the so-called *Erziehungsrenten*; this is a very special case for divorced survivors raising children).

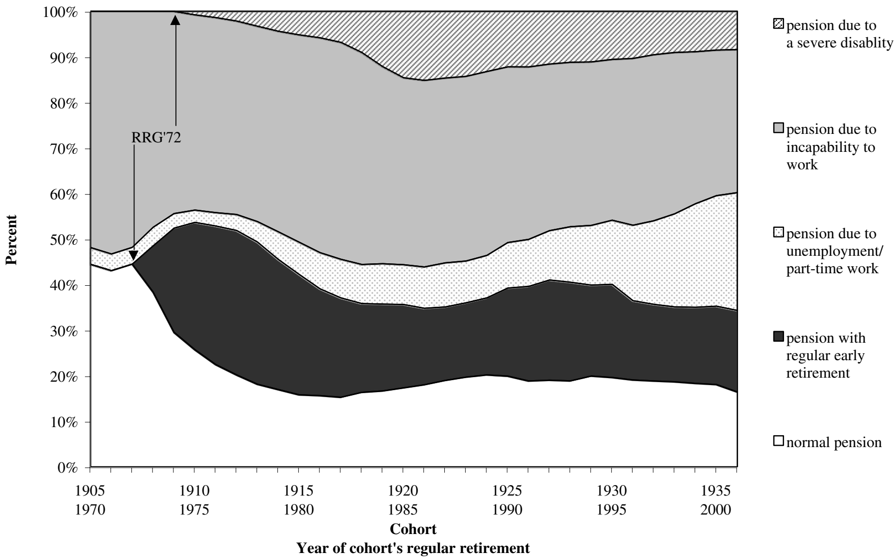
[21] The regular retirement age will be 63 for pensions due to incapability to work or disability and 65 otherwise.

[22] In our benchmark case, the reduction of benefits due to working one year less at the average income equals $\frac{1}{45} \approx 2.2\%$.

Table 6.4. Different paths of entering retirement and conditions that have to be met

		58	59	60	61	62	63	64	65	66	67
	3 years of covered work in the last 5 years, 5 years of coverage	*Benefits due to reduced capacity to work / occupational incapacity* *(Rente wegen verminderter Erwerbsfähigkeit)*								Benefits due to age (Altersrente)	
	8 years of covered work in the last 10 years, 15 years of coverage	*unemployment*			*Benefits due to age after unemplyoment and old-age part-time employment* *(Altersrente nach Arbeitslosigkeit und Altersteilzeitarbeit)*						
	8 years of covered work in the last 10 years, 15 years of coverage		*old-age-part-time-employment* *(Altersteilzeitarbeit)*		*Benefits due to age after unemplyoment and old-age part-time employment* *(Altersrente nach Arbeitslosigkeit und Altersteilzeitarbeit)*						
	10 years of covered work after age 40, 15 years of coverage				*Benefits due to age for women* *(Altersrente für Frauen)*						
	35 years of coverage and severely handicapped (at least 50 percent)				*Benefits due to old age for severely handicapped persons* *(Altersrente für Schwerbehindert)*						
Scenario 2 and 3	35 years of coverage						*Benefits due to age for long-term contribution-payers* *(Altersrente für langjährig Versicherte)*				
Scenario 1	5 years of coverage									Benefits due to age (Altersrente)	

Source: Arnds and Bonin (2002).



Notes: Male retirees, without survivor benefits.
Source: VDR (2002a), author's presentation.

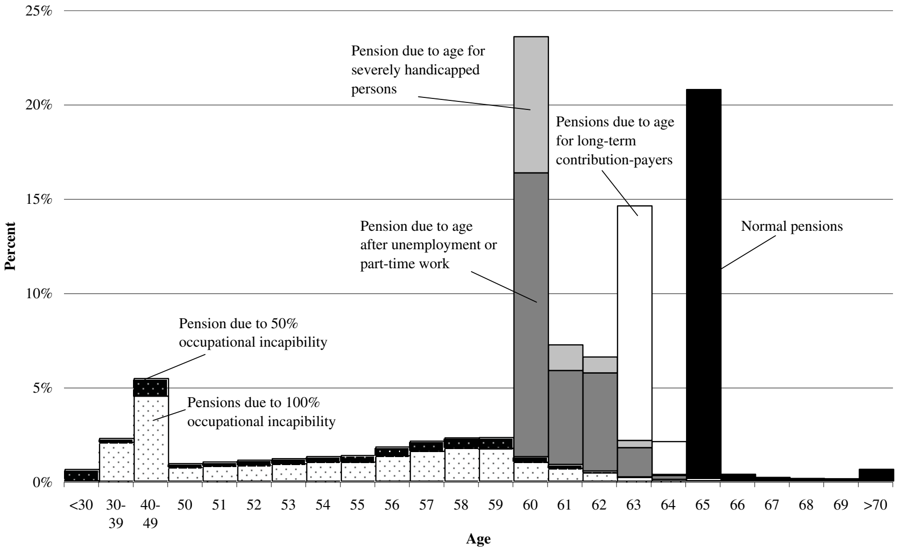
Fig. 6.1. Ways into retirement by cohort in Germany

To conclude this section, we illustrate how the different pension types have been made use of over time. In Fig. 6.1, we depict how many percent of each male cohort have entered retirement via the different pension types. The types are sorted by the degrees of freedom the retiree has in choosing his retirement. The top two pension types (disability and incapability) are pensions that are (at least formally) dependent on the medical condition of the retiree. The path into retirement after part-time work is mostly in accordance with the employee. Entering retirement after unemployment is much less a decision that is made by the employee. However, there is an indication that also unemployment is chosen very deliberately as a path into retirement.[23] Finally, the two bottom types of pensions are those that mark the choices of entering retirement normally, that is, without any conditions on things that are not completely in the control of the retiree.[24]

Figure 6.1 indicates that the pensions for severely handicapped persons have partially crowded out the pensions due to incapability to work, and the pension under regular early retirement have substantially reduced the share of people that start receiving benefits at the regular retirement age for all cohorts entering retirement after 1972. Because of the generous rules for pensions after part-time work, the share of this type of pension has rapidly increased to over 20 percent, reducing all other types

[23]Many employees are entering unemployment at exactly that age that will give them unabridged unemployment benefits just until the age of being eligible for pensions.

[24]Most people who are retiring at the regular retirement age are not retiring earlier because they do not fulfill the requirement of 35 years of service.



Notes: Male retirees, without survivor benefits.
Source: VDR (2002b)

Fig. 6.2. Age and path of entering retirement in 2001

of pensions. In addition, the current distribution of types of pension and at what age they are chosen is displayed in Fig. 6.2.

In the following sections, we will only concentrate on the generosity of the bottom two types of pensions of Fig. 6.1. This is certainly a restriction since only about one third of all individuals are entering retirement via these two channels. We do, however, put up with this loss of generality for several reasons. First and most importantly, it is close to impossible to track all these different types of pensions into one consistent measure of generosity. Secondly, the qualitative changes to the generosity have followed a quite similar path for all types of pensions except for that of pensions after part-time work. This last exemption is therefore the most severe set-back to our approach. However, the pension after part-time work is as much a labor-market policy and a subvention of firms as it is a policy towards the aged.

6.2 The Volatility of Individual Social Security Wealth: A Cohort Analysis of the Generosity of the Benefit Rule

After having worked our way half-decent through the jungle of nearly 50 years of social security reforms, we can finally begin to measure the generosity of the benefit rule over time. In order to do so, we first describe how we construct our measure of relative generosity of the benefit rule in Germany and will then proceed to apply this indicator to the three cohorts born in 1930, 1940, and 1950 for three different scenarios, respectively.

All three scenarios considered are based on the standard retiree. We further assume for all three scenarios that the person is male, single, and without children. The difference between the scenarios are in concern to whether the person is making use of the early retirement option (scenarios 2 and 3), and whether he has spent time in education (scenario 3). Scenario 1 is hence a standard retiree who has not spent time in education after the age of 16 and is retiring at 65. Scenario 2 differs to the first scenario in regard to the timing of retirement: independent of the size of the adjustment factor, the person will always choose to retire at the earliest age possible under the rules for regular early retirement.[25] Finally, under scenario 3 all assumptions of scenario 2 apply, and in addition the person under consideration has spent seven years in education after the age of sixteen. To ease notation, we will give names to the three scenarios: STAN, EARLY, and EARLY ED for scenarios 1 through 3 respectively. While all three scenarios are rather unrealistic, not all restrictions that result from these assumptions are a major set-back for our purposes. Because we are interested in the changes to the generosity of the benefit rule over time, the assumptions only become important when they are touched by changes to the benefit rule. In Sect. 6.2.3, we discuss the restrictions of our approach.

Finally, before we commence with our analysis, it is helpful to stress what we are **not** trying to investigate with our measure of relative generosity. For now, we are neither interested in the sustainability of the public pension scheme, nor are we interested in questions of intergenerational distribution. Both of these questions have already been covered extensively: numerous official and unofficial studies have presented projections of the social security contribution rate addressing issues of sustainability.[26] Schnabel (1998) calculates the rates of return of the public pension system for different cohorts, and he finds that the returns are continuously decreasing the later a cohort is born. Depending on the scenario under consideration, cohorts born after 1960 may actually be facing negative rates of return.[27] Finally, the method of generational accounting has addressed both questions, sustainabilty and intergenerational distribution, at the same time. In Sect. 6.5, we apply the method of generational accounting to assess the sustainability of the legal status quo of the social security system.

6.2.1 A Measure of Relative Generosity

We construct a measure of relative generosity that is based on gross social security wealth (SSW).[28] The SSW, as defined in Eq. (6.1), is the sum over all benefit

[25] Choosing early retirement will of course reduce personal eligibility to 43 EP for the cohorts born in 1930 and 1940 and to 42 EP for the 1950-cohort.

[26] See Sinn and Thum (1999) for a survey of different contribution-rate-projections and the influence of different assumptions on these projections. Werding (2000) analyzes the influence of the RRG92, the RRG99, and a preliminary version of the 2001 reform on future contribution rates.

[27] Eitenmüller (1996) and Hain et al. (1997) conduct similar studies. In contrast to Schnabel, they compute nominal rates of return.

[28] The calculation of social security wealth has first been introduced by Feldstein (1974).

payments from the time of entering retirement until the expected age of death. We adjust the nominal payments with the consumer price index, and we discount them to a specific point in time. The SSW_t in a given year t is equal to the discounted sum of future real benefit payments conditional on the policy that is valid in period t. We also take account of transition periods and protection of trust rules by specifically applying the policy that will be valid in a future year i as specified under the laws in t. Note that all three components of the benefit formula described above are conditional on the policy in t and also the age of retirement may vary depending on the scenarios and the policy in t for the cohort under consideration.

For our analysis, it is important that the value of SSW does not rise only due to moving one year closer to retirement. Therefore, we have to discount the stream of benefit payments to one specific age of the retiree instead of taking the present value at every point in time. A natural choice for this date would be the age of entering retirement (RA). However, since the retirement age varies between scenarios and cohorts, D^c will not always be identical to $RA_i^{c,s}$. Instead, we choose one discounting-year for all scenarios of one cohort. The age that we discount to equals the earliest possible retirement age under the considered pension types and scenarios. Because the pension reform from 1997 allows the cohort born in 1950 to retire at the age of 62, we choose $D^{1950} = 1950 + 62 = 2012$. In order to guarantee the comparability between the three cohorts, we calculate the discounting date for all three cohorts by $D^c = c + 62$.

$$SSW_t^{c,s} \equiv \sum_{i=D^c}^{c+65+L_{65}^c} \frac{B_i(EP_i^{c,s}, ADJ_i^{c,s}, V_i, RA_i^{c,s} | P_t^i)}{CPI_i} \cdot R^{-(i-D^c)} \qquad (6.1)$$

with

$SSW_t^{c,s}$	gross social security wealth at time t of cohort c under scenario s
c	year of birth of a specific cohort
s	scenario
D^c	date to which benefits are discounted for cohort c
L_{65}^c	remaining life-expectancy conditional on reaching age 65
$B_i^{c,s}$	benefit payment in period i conditional on the effective policy at time t
$EP_i^{c,s}$	years of coverage scaled by average earnings (determine benefit entitlement)
$ADJ_i^{c,s}$	adjustment factor for different types of pensions or reductions for early retirement
V_i	value of pension entitlement at date i (indexation)
$RA^{c,s}$	age of retirement of cohort c under scenario s
P_t^i	policy of benefit payments as legislated at date t for benefit payments in year i
CPI_i	consumer price index for year i
R	gross interest rate $(1 + r)$, with r = real interest rate

In order to calculate the SSW, we need to make assumptions on the future development of parameters that determine the pension payments. In detail, we assume that both the real growth rate and the rate of inflation after the year 2002 will be at one percent per annum, respectively. We discount all pension payments by three percent per annum. Further, we assume that the retirees share of health insurance contributions will be constant at 7.85 percent from 2003 onwards. Also, the quota of net-to-gross-earnings and the quota of net-to-gross-pensions (cf. Table 6.3) will remain constant after 2001 at 65.4 percent and 92.4 percent respectively. The remaining life-expectancy of a 65-year-old is assumed to rise from 16 years in 2001 to 22 years in 2034. The projection of the contribution rate to the public pension system is taken from Birg and Börsch-Supan (1999), according to which the contribution rate will rise to 24 percent.

Since we are primarily interested in the *changes* to the benefit rule, the calculation of the SSW is only an intermediate step in calculating our measure of relative generosity. This measure is based on using the benefit formula as it is effective in a respective year and assuming that it will remain valid from then on. Applying this principle, we calculate the SSW at the time of entering retirement for each year from 1970 until 2002. The relative generosity of the pension scheme is then calculated by the fraction of SSW_t of each respective year t over the social security wealth that would have resulted if the laws concerning the benefit rule from 1970 still were effective.

$$RGC_t^{c,s} \equiv \frac{SSW_t^{c,s}}{SSW_{1970}^{c,s}} \tag{6.2}$$

By constructing this cohort specific measure of relative generosity (RGC), we show how the generosity of the German public pension scheme has increased or decreased over time. By definition, the value of RGC_{1970} equals one. The measure, therefore, shows by how many percent the generosity of the laws at every point in time t deviates from the generosity under the laws from 1970.

6.2.2 The Impact of Benefit Rule Changes on Relative Generosity for Selective Cohorts

We calculate our measure of relative generosity for cohorts (RGC) given in Eq. (6.2) for the three cohorts c born in 1930, 1940, and 1950. For each cohort, we consider the three earlier described scenarios s: the standard pensioner (STAN), the early retiring standard pensioner (EARLY), and the early retiring pensioner with education (EARLY ED). We display the results in Fig. 6.3. Figures 6.3a)-c) show all three scenarios for each respective cohort. In Figures 6.3d)-f), we present a single scenario for all three cohorts in each respective sub-figure. We have chosen a logarithmic scaling of the vertical axis in order to illustrate the size of the relative changes between years at a later point in time.[29] As expected from the qualitative analysis of changes over time, summarized in Table 6.2, the generosity has increased in 1972 under all scenarios and for all cohorts, but has steadily fallen since. One can follow the impact of the different changes to generosity by comparing Fig. 6.3 with Table 6.2. Changes of the

[29] If we did not use a logarithmic scaling, later reforms could not be well compared to the laws that were valid just before the last reform, because the changes are all in relation to the laws of 1970. The logarithmic scaling avoids this problem, since the graphical representation is in line with the period-to-period change of the generosity.

generosity under scenario STAN are all due to alterations of the indexation of the benefits and due to the introduction of health care contributions (columns 8 through 11 in Table 6.2). The differences between the scenarios STAN and EARLY can also be traced by focusing on the changes listed in column 5 of Table 6.2: EARLY shows a sharp increase of generosity in 1972 with the introduction of the early retirement option. For cohorts 1930 and 1940, the relative generosity to STAN and EARLY will run in parallel paths after 1972 with EARLY being roughly 10 percentage points above STAN. For the cohort born in 1950, scenario EARLY will experience a sharp drop in generosity in 1989 when the adjustment factor ZF was introduced for future early retirees.[30] In scenario EARLY ED, the changes to personal eligibility due to educational years significantly reduce the relative generosity in addition to scenario EARLY. This large drop in generosity is not surprising, since personal entitlements due to education were reduced from the equivalence of 14 EP in 1970 to 2.25 EP today under scenario EARLY ED.[31]

In addition to Fig. 6.3, we summarize the maximum and minimum levels of relative generosity in Table 6.5. Obviously, the losses and the volatility of relative generosity are largest for scenario EARLY ED. The standard retiree – STAN – is the scenario with the lowest volatility in relative generosity, even though the level of relative generosity is higher under scenario EARLY. As can be seen from Table 6.5, the relative differences between the maximum values of relative generosity in year 1972 and the lowest values in year 2001 amount to losses that vary – depending on the considered scenario and cohort – between 32 and 57 percent. For the 1930 cohort, the relative generosity has dropped from its highest level to its lowest level 33 percent under scenario STAN and 43 percent under scenario EARLY ED. Under scenario EARLY, the relative drop has only been 32 percent with a standard deviation for RGC^{1930} of 0.13. For the other two cohorts under consideration, the relative difference under scenario EARLY amounts to a drop of relative generosity of 37 and 40 percent for cohorts 1940 and 1950 respectively. The highest reduction of generosity is that of cohort 1950 under scenario EARLY ED. The standard deviation in this case is over 20 percent.

[30] The 1997 increase of generosity that allowed early retirement with deductions at 62 instead of 63 is not very large and can hardly be seen in Fig. 6.3c).

[31] Actually, we put a cap on the maximum loss by assuming only seven years of education under scenario EARLY ED. The maximum amount of years that were accredited in 1970 was 13 years which led to an entitlement of 26 years of contribution payments at the average salary.

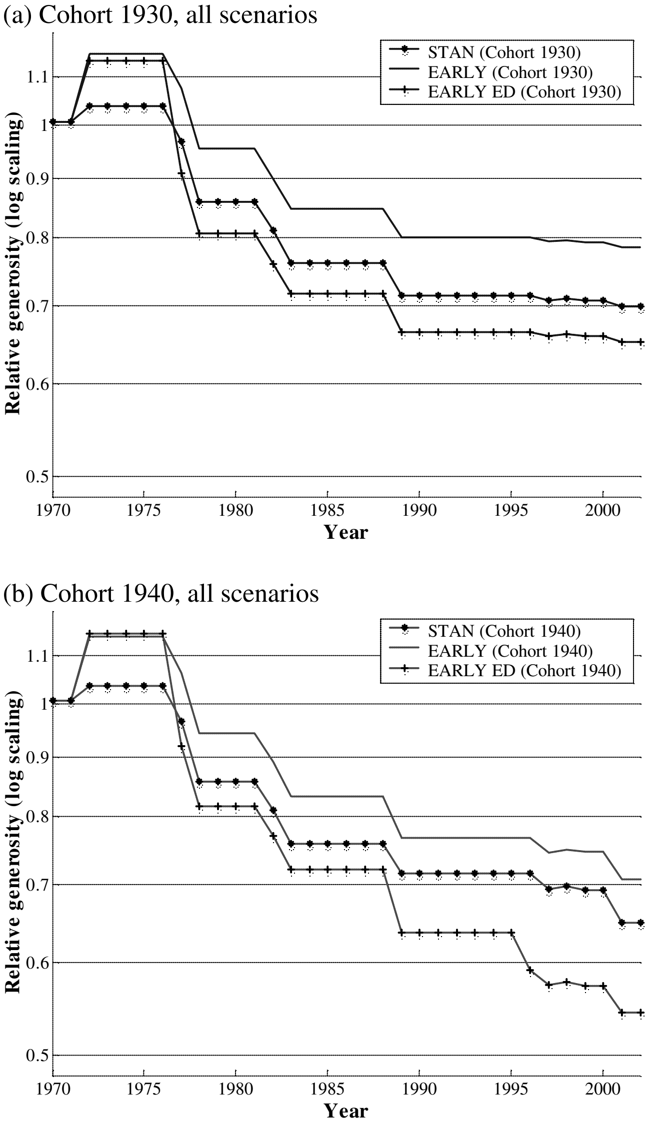
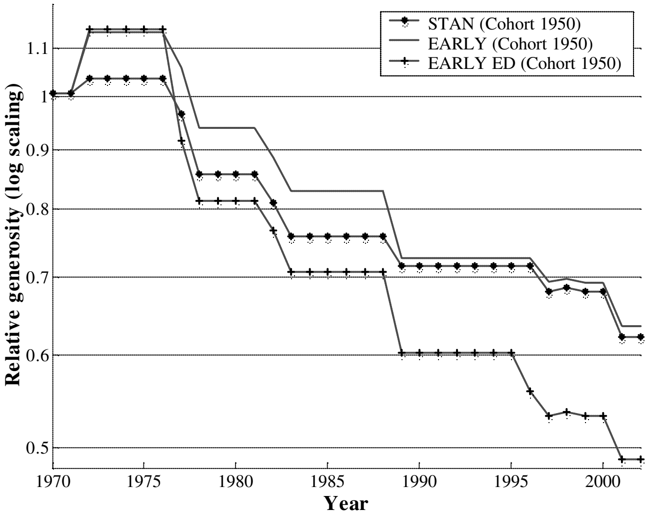


Fig. 6.3. Relative generosity for cohorts 1930, 1940, and 1950 (RGC)

(c) Cohort 1950, all scenarios



(d) Scenario 1, all cohorts

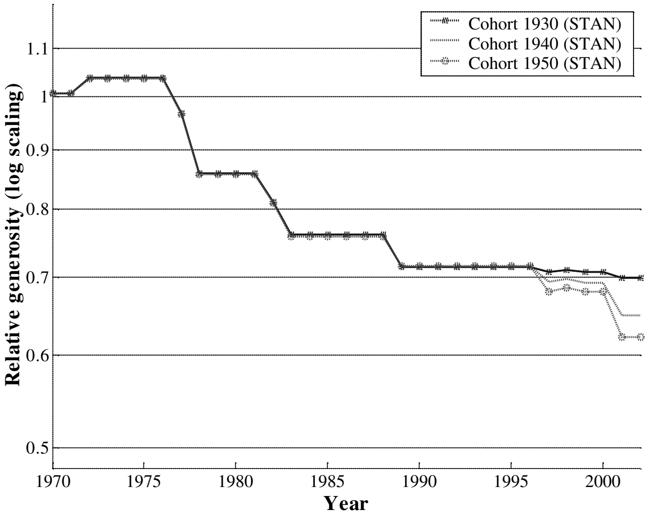
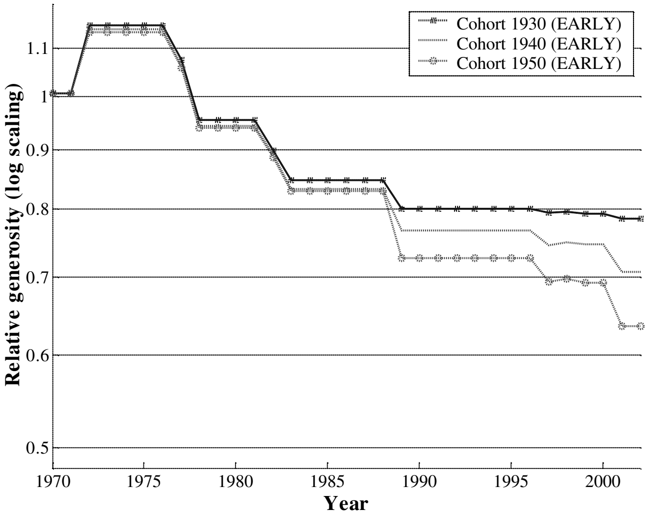


Fig. 6.3. (Continued)

(e) Scenario 2, all cohorts



(f) Scenario 3, all cohorts

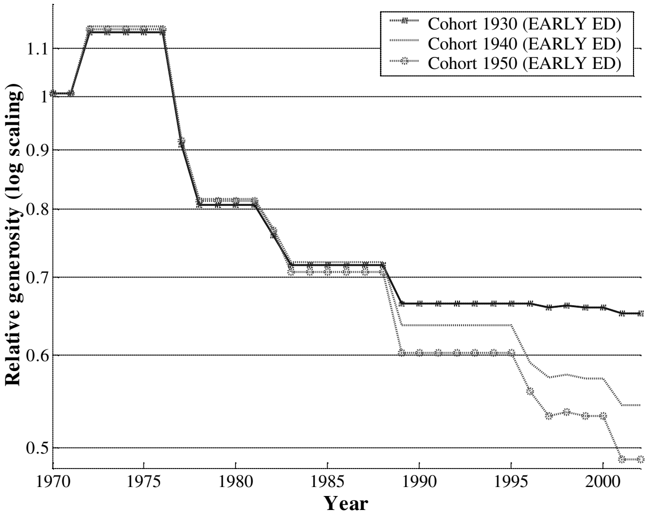


Fig. 6.3. (Continued)

It is noteworthy that the relative losses tend to vary more between scenarios for one cohort than between cohorts for one scenario. Also, for most cases under scenario STAN, the path of relative generosity is quite similar for all cohorts. Only the two most recent major reforms of the dynamic component of the benefit formula discriminated between the different cohorts (see Fig. 6.3d). In fact, this equal treatment of cohorts holds quite general for all changes to the indexation of the benefit formula. Reforms of the early retirement options, especially those of the RRG92, are, on the other hand, highly discriminating in their treatment of different cohorts.

Finally, it also comes as a surprise that among the changes to the indexation of the benefits, even seemingly minor changes to the statistical calculation of benefits may have quite a significant impact on relative generosity. This can be seen by the rather large reduction of relative generosity in 1977 and 1978 under scenario STAN.[32]

Table 6.5. Difference between maximum and minimum values of relative generosity

	Cohort 1930		
	STAN	EARLY	EARLY ED
Max	1.03	1.14	1.13
Min	0.69	0.78	0.61
Difference	32.7%	31.8%	45.7%
Standard deviation	12.8%	13.0%	18.0%
	Cohort 1940		
	STAN	EARLY	EARLY ED
Max	1.03	1.14	1.14
Min	0.64	0.70	0.54
Difference	37.4%	38.2%	52.7%
Standard deviation	13.1%	14.3%	19.8%
	Cohort 1950		
	STAN	EARLY	EARLY ED
Max	1.03	1.13	1.13
Min	0.62	0.63	0.48
Difference	40.0%	44.0%	57.3%
Standard deviation	13.3%	15.9%	21.1%

[32] In Appendix A.6, we show that the change in the calculation of AB will lead to a reduction of generosity of the benefit formula if the average growth rate during pension payment is less than 8.3 percent. In comparison, the average growth rate since 1977 has been 3.5 percent with even much lower values in the past decade.

6.2.3 Discussion and Caveats

Our analysis has shown that the relative generosity, and therefore the gross social security wealth, has been drastically reduced over the past three decades. From the way we have constructed our measure of relative generosity, political changes of the benefit formula are the only source of risk that we consider in our analysis of the German public pension scheme. Our results show that the standard deviation of RGC under all considered scenarios and for all cohorts is well over 10 percent, and the gross social security wealth can be subject to losses between 30 and 57 percent over the life-cycle. We will now touch on some caveats of our approach:

i) Diamond (2002, p.46) has rightfully pointed out that "...it is not clear whether the possibility of further legislation should be viewed as a political risk or a political hedge. The possibility of adapting social security to changing economic and demographic circumstances makes it more valuable to society, not less valuable." Still, we feel that our analysis is worthwhile for several reasons. First, the magnitude of these changes is nevertheless interesting. Second, we are interested in whether the elderly are generally protected from social security reform or not. Finally, we also try to identify the influence of demographic developments on our measure of generosity (cf. Sect. 6.4).

ii) Our perspective is of course restrictive, since we assume that individuals are foresighted enough to include their social security wealth in the calculation of their life-cycle resources, but are at the same time naive enough to believe that the current law will still be valid for them. There is some evidence that individuals do have quite realistic expectations on future social security reform. According to a recent survey of opinion conducted by Boeri et al. (2001b), 75 percent of the German population are expecting that in the course of the next 10 years there will be a reform reducing significantly the level of the public pension. On the other hand, the government tends to induce the believe amongst individuals that the current benefit formula will be valid at later times. For example, the most recent reform (AVmEG) obliges the administration of the pension scheme to provide information of personal eligibility for current contribution payers. This information is similar to our values of SSW since it will provide the working population with information on the prospective future size of benefit payments under current law.

iii) As stressed by Schnabel (1998) in his calculation of rates of return and touched upon by McHale (2001) in an appendix, the expected SSW of someone facing a certain expected remaining life-time may deviate from the SSW of someone with an uncertain remaining life-time. This is due to the familiar result from Jensen's inequality that the expectation of non-linear functions evaluated at the random variable is not equal to the function evaluated at the expectation of the random variable. McHale (2001) argues that for the calculation of SSW there will be both a downward bias, because of the uncertainty of reaching retirement, and an upward bias during pay-out, if the interest rate is larger than the growth rate. The sign of the bias is therefore undetermined. Because our measure is a relative one that has this potential bias both in the nominator and denominator, it is highly unlikely that the simplification of using a certain life-time will have severe consequences. Only reforms that change the benefit payments with a non-uniform time-profile for a single cohort may lead to a potential bias.

iv) Because we are only looking at the gross social security, we do not need to consider the timing and size of contribution payments. The age-profile of earnings is not important because the German public pension system does not use a sub-sample of past earnings to calculate eligibility and because we are not analyzing the implicit rate of return of the contributions to the scheme. A calculation of the rates of return of the pension scheme would also have to consider the effects of each reform on the future development of the contribution rate.

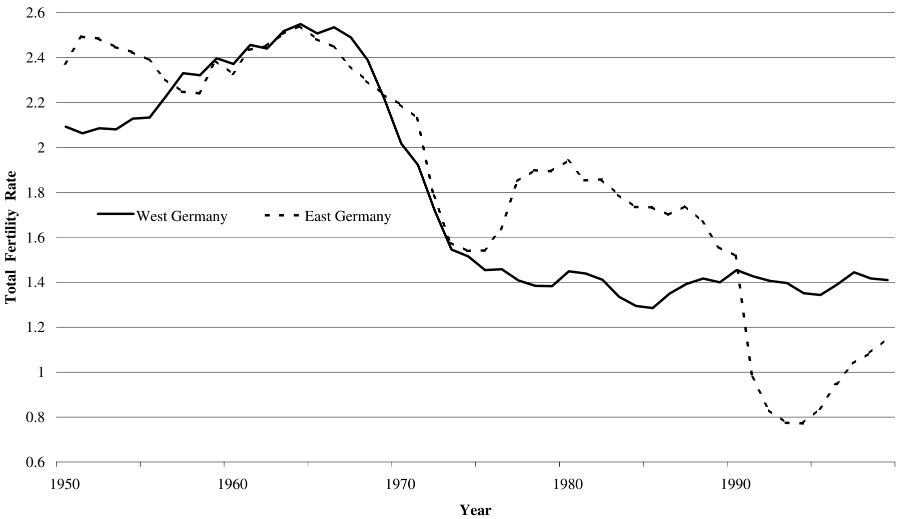
v) Finally, there are restrictions due to the choice of the scenarios. By only considering standard retirement and regular early retirement, we neglect that there are a number of other paths into retirement and that they play a major role in the German pension system as presented in Sect. 6.1.4. Furthermore, we are only looking at the rules for male retirees. This is certainly a set-back, but we feel that creating even more scenarios will not yield significant different results because the general direction of changes is similar for the cases we have not considered as for those we did consider. Nevertheless, two restrictions apply: contrary to the scenarios we are analyzing, pensions after part-time work and individual eligibility due to time spent rearing children were both increased during the 1980s and 1990s. Not including entitlements for rearing children is, however, offset by the fact that women face even larger reductions in generosity then men do. This is because the cut-back in generosity under the early retirement option has been more severe for females. Also, we do not account for changes in the rules concerning the benefit payments to dependents.

6.3 Demographics and Population Projections

In this section, we first draw a general picture of the demographic developments specifically for Germany; cf. Sect. 3.1 for an international perspective of the aging process. We then touch upon past population projections, and we show how well these projections performed at forecasting the old-age dependency ratio. We will argue that since the late seventies, the official population projections did a good job at predicting the old-age dependency ratio at a range of roughly 15 years and that the demographic development and its influence on social security has found its way into the political debate since the mid-seventies. This is important for the analysis we conduct in Sect. 6.4, where we use future values of the old-age dependency ratio as a determinant for the generosity of the pension scheme.

6.3.1 Demographic Developments of the Past Decades

In Germany, like in many other developed countries, a demographic transition towards an aging population is currently taking place. Today, the population pyramid, whose



Notes: East German figures from 1950 through 1954 do not take into account East Berlin
Source: Statistical Yearbook of the Federal Republic of Germany, Statistical Yearbook of the German Democratic Republic, several years

Fig. 6.4. Birth rates for East and West Germany 1950–1999

name originates from the silhouette of the graphical representation of the population structure, resembles much more the contour of a mushroom than that of a pyramid. The causes for this development can be identified in the extreme changes of the fertility rates and in the continuously increasing life-expectancy.

As one can see in Fig. 6.4, the total fertility rates in East and West Germany have undergone a sharp increase in the late fifties of the last century and stayed at a level around 2.5 for roughly ten years. The cohorts born during this time – generally referred to as the Baby-Boomers – were immediately followed by cohort-sizes that were well below the level of reproduction. Since the beginning of the seventies, the total fertility rate has stabilized at around 1.4 in West Germany, which is about 30 percent short of reproduction. Because of this immediate succession of very high and very low birth rates one sometimes speaks of a "baby-boom baby-bust" scenario.

At the same time, the life-expectancy has increased over the past century at an afore unknown speed. Except for the time span between 1960 and 1970, where the increases were quite moderate, life-expectancy at birth has consistently increased every ten years by approximately three years. In Table 6.6 we also show how the remaining life expectancy conditional on reaching the age of 65 has changed over the years. Obviously this value is of great interest for matters concerning social security, since it is an indicator of the duration of benefit payments. This remaining life-expectancy has risen by over 50 percent for men and nearly doubled for women. At the same time the probability of reaching the age of 65 has increased from 25 percent to 80 percent for men and from 30 percent to 90 percent for women.

Table 6.6. Life-expectancy in Germany

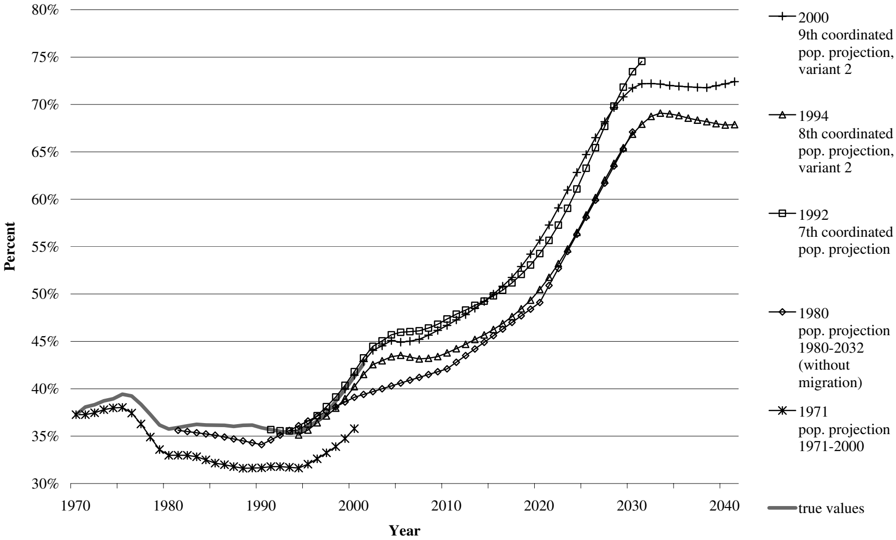
mortality table	life-expectancy			
	men		women	
	at birth	with 65	at birth	with 65
1871/1881	35.58	9.55	38.45	9.96
1891/1900	40.56	10.12	43.97	10.62
1901/1910	44.82	10.40	48.33	11.09
1910/1911	47.41	10.38	50.68	11.03
1924/1926	55.97	11.46	58.82	12.17
1932/1934	59.86	11.87	62.81	12.60
1949/1951	64.56	12.84	68.48	13.72
1960/1962	66.86	12.36	72.39	14.60
1970/1972	67.41	12.06	73.83	15.18
1986/1988	71.70	13.77	78.03	17.30
1995/1997	73.62	14.91	79.98	18.66
1996/1998	74.04	15.13	80.27	18.85
1997/1999	74.44	15.36	80.57	19.06

Notes: Until 1932/34 Reichsgebiet, 1949/51 former FRG without Saarland and Berlin, 1960/62 and 1970/72 former FRG, from 1986/88 Germany.
Source: Statistical Yearbook of Germany, several years.

We depict the old-age dependency ratio in Fig. 6.5. Here, the old-age dependency ratio is defined as the proportion of the population that is 60 years and above in relation to the 20 to 59 year old population. The most recent official population projection forecasts roughly a doubling of this ratio within the next thirty years (see Fig. 6.5).

6.3.2 On the Precision of Past Population Projections

Keeping track of the size and structure of a countries population by conducting a population census has been a common practice for over two millenniums. Using the data collected in such a census together with mathematical models to project future developments of the population is, compared to the over two thousand year long history of the census, a very recent undertaking. However, enough time has gone by since the introduction of the formal framework by Leslie (1945) that by now we have some official projections to look back upon and evaluate their performance at predicting future demographic developments. In Fig. 6.5, we plot the old-age dependency ratio over time as predicted by various official population projections. In addition, we include the true figures of the old-age dependency ratio from 1970 through 2001. Inspection of this figure shows that only the 1970 projection was really off the mark. Apparently, the decelerated increase of life expectancy of the 1960s led to a misjudgement concerning the future rise of life-expectancy. The rise in longevity was again underestimated in the projection of 1980, however not as severely as in 1970. Also, because birth rates were correctly assumed to stay at the low level of the seventies, the long-run values of the old-age dependency ratio correspond to those of the more



Notes: Old-age dependency ratio at the beginning of each year is defined as $\frac{60+}{20-59}$. Except for the 8th and 9th coordinated population projection, the values are for the Old Laender.
Source: Statistisches Bundesamt (1971), BMI (1984), Sommer (1992,1994,2001) and Statistical Yearbook of Germany, several years.

Fig. 6.5. Old-age dependency ratio 1970–2042: true values and projections

recent projections. The 1992 projection shows how good a population projection can perform in the medium term.

From Fig. 6.5, we conclude two things. Firstly, an idea of the extent of the population crises has been around since the late seventies and was officially documented with the population projection of 1980. Secondly, at the medium term of 15 to 20 years, the past population projections were quite precise at predicting the level of the old-age dependency ratio. These findings are important for our purposes, since we will analyze whether political changes to the benefit formula can be explained by lead values of the old-age dependency ratio. In order to conduct this analysis, it is a prerequisite that these projections exist, that they forecasted a "demographic crisis", and that they are fairly precise at the medium term.

The implications of these projections have also found early access into the political debate on the future of the public pension scheme. Grohmann (1980) conducted a study for the federal ministry of labor and social security where he finds that projections with a horizon of 15 years ahead are useful for the analysis of the financial situation of the *GRV*. A federal institute for the research on demographics (*BiB*) was founded in 1973 as a think tank of the political implications of demographic changes. A report on the research conducted at the *BiB* during the first 25 years of its existence can be found in Höhn (1998). A comprehensive survey on the future of the public pension in an aging society can be found in Eekhoff (1985).[33]

6.4 Demographic Development and Benefit Rule Changes

In this section, we try to estimate what the driving forces are behind changes to the generosity of the public pension system. Specifically, we investigate how current and future values of the old-age dependency ratios can explain the changes to the prospective generosity of the pension scheme for a middle aged worker and the changes to the generosity for a person who is just about to enter retirement. This analysis is in the spirit of McHale (2001) and Razin et al. (2002). The former author analyzes how the fraction of old-age-expenditures to GDP can be explained by the old-age dependency ratio. A regression for the OECD countries yields the result that a relative increase of the old-age dependency ratio increases the share of old-age-expenditures to GDP by roughly 1.6. This elasticity is larger than unity and indicates that per-capita transfers to the old are actually increased during demographic transition.[34] McHale gathers further data on recent reforms in the G7 countries, and he comes to the conclusion that the currently old are protected from severe social security reforms. The middle aged are, however, subject to cuts in generosity. He formulates a model of political economy to explain this by introducing the fear of the middle aged that future generations might abolish the system altogether.

Razin et al. (2002) take a broader look at the size of the welfare state and find that the elasticity of social transfers with respect to the dependency-ratio equals -7.5. They put forward a model that can explain a shrinking welfare state in an aging economy if the median voter is not yet part of the old population. This is due to the increased fiscal “leakage” for the median voter in an aging society. Because of the rising share of transfers that are paid to the elderly, the low-skilled median voter can turn from a net-beneficiary to a net-contributor during demographic transition, and he will then be in favor of reducing the welfare state.

Our analysis deviates from both of these studies for two reasons: first, we are not estimating an actual time-series of transfer payments, but rather a measure of relative generosity that we construct from the laws of the benefit formula. This allows us to include reforms that are only gradually phased in and that are therefore not included in actual data yet, or are not visible in the data at the time the reform is made. Second, we add the hypothesis that future values of the old-age dependency ratio may have an influence on the generosity of the pension system.

[33] Surprisingly (or not), the different proposed measures to reform the public pension scheme have not changed much since then.

[34] The inclusion of country dummies reduces this elasticity to 0.2. Unfortunately, McHale does not comment much on this, even though this drastically changes his results. A value of 0.2 implies a severe cut-back of per-capita-transfers instead of an increase.

6.4.1 Time–Series Analysis of Relative Generosity

In order to conduct our analysis, we first need to modify the measure of relative generosity to be applicable over time. We do so by calculating the relative generosity over time (RGT) for persons of a specific age at each point in time. In particular, we calculate the gross social security wealth of a 45 and 62 year old under the applicable law in year t. Again, we compare this value of SSW_t to the social security wealth of the respective cohorts under 1970 law in order to derive our measure of relative generosity.

$$RGT_t^{45} = \frac{SSW_t^{c=t-45,s=2}}{SSW_{1970}^{c=t-45,s=2}} \quad \text{and} \quad RGT_t^{62} = \frac{SSW_t^{c=t-62,s=2}}{SSW_{1970}^{c=t-62,s=2}} \qquad (6.3)$$

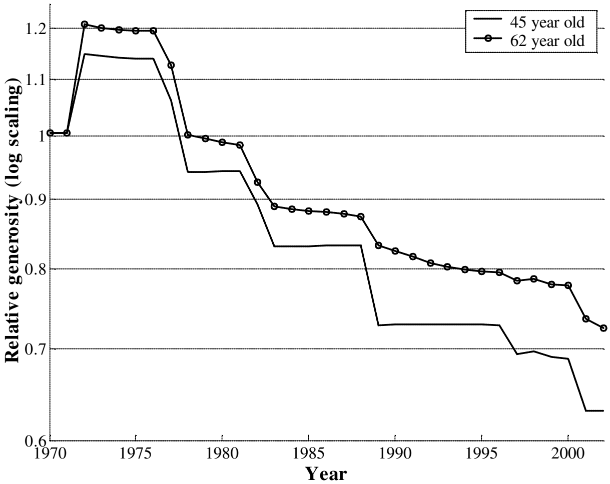
In this section, we are concentrating on scenario 2 only, i.e. the standard retiree without education who is retiring early. We do so because we want to keep the number of regressions to a limited amount. We choose scenario EARLY because we believe this is the most representative scenario for two reasons: not only do most people who meet the necessary requirements choose the early retirement option, but also the time path of generosity for early retirement is quite similar to that of the pension due to occupational incapacity.

The time path of RGT^{45} and RGT^{62} is plotted in Fig. 6.6. Because we are looking at 45 and 62 year old persons at every point in time, we are considering a different set of two generations in every year. This inclusion of many cohorts in one indicator raises the question of how we treat the rising remaining life-expectancy over time. For the regression of the policy function, we use a constant value for the remaining life-expectancy.[35] We proceed in this manner because we want the demographic variable to be an explaining variable and not be part of the explained variable. In Fig. 6.6, we have, however, also included a sub-figure in which we illustrate the development of generosity if we take account of rising life-expectancy.

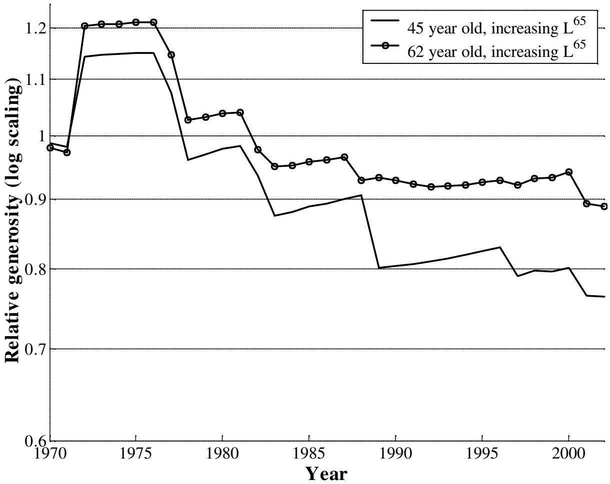
The time-path of RGT^{45} is almost identical to that of cohort 1950 under scenario EARLY (cf. Fig. 6.3c). The time path of RGT^{62} differs to that of RGT^{45} with respect to two features. Firstly, the increase of generosity in 1972 is larger for RGT^{62} because the average remaining life-expectancy we use for RGT^{62} is smaller than that for RGT^{45}. Therefore the relative increase of adding two more years of pension payments is larger. Secondly, the time path of RGT^{62} exhibits a more continuous course of changes in contrast to the discrete jumps in the time path of RGT^{45}. The explanation for this is that some measures are only implemented gradually. A good example is the introduction of health care contribution payments for retirees in 1983. Because the cohorts entering retirement in 1984 and 1985 are also beneficiaries during years with a reduced contribution rate, they are not hit by the full extent of the reform. However, as one can see from Fig. 6.6a, the entering retirees are by no means seriously protected from benefit rule changes.

[35] To be precise, for RGT^{62} we have taken the average remaining life-expectancy of the 65 year old male of all generations that are considered in this scenario. For RGT^{45}, we proceed in the same way. The resulting figure is higher for RGT^{45} because the sample of cohorts used to calculate this average is born seventeen years later than in the case of RGT^{62}.

(a) with constant life-expectancy



(b) with increasing life-expectancy



Source: Author's calculations

Fig. 6.6. Relative generosity for the 45 and 62 year-old (RGT) from 1970 through 2002

6.4.2 Estimating a Policy Function of Generosity: Does Germany Have an Implicit Demographic Factor?

In the above mentioned survey of opinion conducted by Boeri et al. (2001b), the majority of the Germans interviewed were in favor of opting out of the public pension scheme. At the same time, there was a large consensus that the size of the welfare state should be maintained at the current level. A question that the survey does not address is the preference of the German citizens if they have to choose between either raising contributions in order to keep per-capita benefits constant or reducing per-capita benefits in order to keep the contribution rates constant.[36] However, this is exactly the crucial question for social security during demographic transition. In Sections 6.2 and 6.4.1, we have shown some evidence that the generosity of the pension scheme has been significantly reduced in the course of the past decades indicating that per-capita benefits are not being held at the constant level. We will now estimate the influence of demographic developments on this trend.

We estimate a regression in which the dependent variable RGT^{45} is a function of the current old-age dependency ratio, of the 17-period forecast of the old-age dependency ratio,[37] and of further variables that are indicators of the short-run financial situation of the *GRV*. These are the federal subsidy to the pension scheme as a fraction of total expenditures, the average gross earnings of the labor income, and the reserves of the *GRV* measured in monthly expenditures (SSREV). All data are taken from VDR (2002a). We use a one period lead of the federal subsidy since the federal subsidy is usually determined beforehand by legislation. Note that we take the natural logarithm of the dependent and all independent variables. We estimate this function by least squares with an included AR(1) term. The AR(1) term is added because for some of the regressions we could not reject autocorrelation of the error terms.[38] The same regressions are run on RGT^{62}. The results for the regressions are given in Table 6.7. We denote significance levels by asterisks, and we include the t-statistics in parentheses.

The results in Table 6.7 show highly significant coefficients for both the current and the future old-age dependency ratios under all models. This holds for all regression on RGT^{45} and on RGT^{62}. Of the other potential determinants added to the regression, only the federal subsidy can be found to have a significant (negative) influence on both measures of generosity. The average earnings also have a significant (positive) influence on the generosity to the 62 year old. However, none of the additional determinants changes the size or significance of the coefficients of OAD and $OAD(+17)$ in a fundamental way. The regressions show that a 10 percent increase in the current OAD is associated with a 6 to 7 percent increase in generosity to the middle aged. On the other hand, a future increase in the OAD has a more than one-to-one negative influence on RGT^{45}. The elasticity is smaller than -1.1 in all regressions.

[36] Consulting the web-appendix of the survey shows that the questionnaire only gave a choice of a) reducing taxes and benefits, b) increasing taxes and benefits, or c) maintaining the current level of contributions (cf. Boeri et al. (2001a)).

[37] We use the true values of the old-age dependency ratio in the years 1970 through 2001 and the figures from the ninth coordinated population projection for the future value of the old-age dependency ratio. We use the true value instead of the forecast when this value is available for three reasons. First of all, we try to avoid discrete jumps in the dependency ratio. Secondly, it is hard to say at what time the official population projection was known to the policymaker. And finally, Fig. 6.5 shows that the projections are not too far off the mark.

[38] We perform the Breusch-Godfrey Serial Correlation LM Test for the residuals of the models with AR(1) specification to verify that the H0 of no serial correlation cannot be rejected for these models.

Table 6.7. Determinants of generosity RGT, least squares regression 1970 through 2001

	log(RGT_t^{45})				
	(1)	(2)	(3)	(4)	(5)
log(OAD[a])	0.37*	0.59***	0.64***	0.67***	0.65***
	(1.75)	(3.01)	(3.13)	(3.22)	(3.13)
log(OAD(+17))	-1.33***	-1.17***	-1.28***	-1.28***	-1.27***
	(-17.15)	(-13.53)	(-6.98)	(-6.67)	(-6.86)
log(federal subsidy(+1))		-0.33***	-0.34***	-0.36***	-0.34***
		(-2.96)	(-3.05)	(-3.38)	(-2.97)
log(average earnings)			0.04	0.03	0.05
			(0.76)	(0.59)	(0.79)
log(SSREV[b])				-0.01	
				(-0.54)	
dummy for unification					-0.01
					(-0.22)
AR(1)	0.47**	0.38**	0.36*	0.32	0.34
	(2.41)	(2.05)	(1.77)	(1.42)	(1.70)
Constant	-0.93	-1.11	-1.61	-1.51	-1.62
Adjusted R^2	0.96	0.97	0.97	0.97	0.96
F-Statistic	244.55	228.15	177.65	143.46	142.61

	log(RGT_t^{62})				
	(1)	(2)	(3)	(4)	(5)
log(OAD)	0.47***	0.68***	0.89***	0.88***	0.89***
	(2.99)	(3.53)	(5.77)	(5.69)	(5.89)
log(OAD(+17))	-1.12***	-0.97***	-1.28***	-1.28***	-1.29***
	(15.12)	(-10.94)	(-9.27)	(-8.77)	(-9.12)
log(federal subsidy(+1))		-0.34**	-0.41***	-0.40***	-0.42***
		(-2.36)	(-3.14)	(-3.07)	(-3.21)
log(average earnings)			0.13***	0.14***	0.13**
			(2.98)	(3.05)	(2.62)
log(SSREV(-1))				0.00	
				(0.24)	
dummy for unification					0.01
					(0.56)
AR(1)	0.49*	0.48**	0.18	0.18	0.18
	(1.74)	(2.16)	(0.87)	(0.86)	(0.82)
Constant	-0.58	-0.79	-2.36	-2.39	-2.35
Adjusted R^2	0.95	0.97	0.97	0.97	0.97
F-Statistic	215.45	230.45	212.58	170.63	172.06

t-statistics are in parentheses
significance is denoted by asterisks (*=10%, **=5%, ***=1%)

[a]Old-age dependency ratio: population 60+/ population(20–59).
[b]Reserves of the *GRV* measured in monthly expenditures (*Schwankungsreserve*).

The general direction of this result is also valid for the generosity to the 62 year old. A rise in the current OAD is associated with an increase in generosity, but higher levels of future old-age dependency ratios are associated with an even larger reduction in generosity today. The coefficients of OAD are slightly higher in the regressions on RGT^{62} than those in the regressions on RGT^{45}. We also run the regressions on the contribution rate (results are reported in Table B.3 in the Appendix). However, no significant results across the different model specifications can be achieved. The elasticity of the contribution rate with respect to changes in the current old-age dependency ratio is consistently positive at values ranging between 0.1 and 0.2, but the results on this parameter are all above a significance level of 10 percent.

The results of the regressions show that the medium to long term demographic development has a significant influence on how the legislator is adapting the generosity of the benefit rule. This indicates that the legislator is taking action today to encounter potential future financial problems of the pension scheme due to demographic crisis. It seems that the German benefit rule is subject to an implicit demographic factor. An important result from our analysis is that future demographic developments are effecting the generosity for both future and current retirees. The findings of McHale (2001) that current old are protected from social security reform cannot be confirmed by our data. On the other hand, without long term demographic pressure, the classic theory of political economy of social security would apply: the positive coefficient on the current OAD indicates that the old will use their growing political power to increase the generosity of the pension scheme. To the author's knowledge, none of the existing models of political economy of social security can explain the phenomenon that the current OAD has a positive influence on generosity while at the same time, the $OAD(+17)$ has a negative influence on generosity. We see this as a point of departure for future research.

6.5 Future Changes: What Does It Take To Make the *GRV* Sustainable?

From the projection of the future old-age dependency ratio depicted in Fig. 6.5, it comes as no surprise that the current level of generosity of benefit payments is not sustainable.[39] Accordingly, the German government has put a task force in place – the so-called *Rürup Commission* – that is working out a proposal for yet another reform of the social security system.[40] In the following analysis, however, we will not consider these latest proposals. Instead, we now turn away from specific reforms, and

[39] Sustainability is defined as a state in which the current laws can be maintained *ad infinitum* without a further increase in the contribution rate or a decrease of transfers.

[40] This commission has suggested two measures: an increase of the statutory retirement age by two years to 67 over the next 30 years and a modification of the indexation of benefits that will take account of future demographic developments. The so called "sustainability factor" is similar, but not identical to the "demographic factor" of the RRG99. The now proposed addition to the indexation-formula will adjust the benefits depending on the development

we address the more general question of the magnitude of further changes that will be necessary to make the German public pension program sustainable. In order to put the past changes to the generosity of the benefit rule into context to the still to be expected changes, we compute the sustainability – or more precisely the sustainability gap – of the public pension scheme. We do this by applying the method of generational accounting presented in Sect. 3.2.1 to the isolated sector of the public pension scheme.

6.5.1 Applying Generational Accounting to the Public Pension Scheme

In this section, we describe how the isolated version of generational accounting is conducted. The aim of the method is an empirical indication of life-cycle net taxes according to the status quo of fiscal policy.[41] At the same time, net tax payments by present and future generations are linked by fundamental macroeconomic constraint: over an infinite horizon, the intertemporal budget constraint of the public sector has to be balanced. The intertemporal budget constraint states that the current government debt will have be financed by the present value of the sum of all future primary surpluses. Generational accounting calculates future primary surpluses by combining population projection together with the profiles of the age-distribution of tax and transfer payments derived from panel data. One speaks of a sustainability gap, if the sum of the future primary surpluses does not suffice to finance the government's debt of the baseyear.[42]

Past studies by Besendorfer et al. (1998), Bonin (2001b), and Borgmann et al. (2001) have used the method of generational accounting to analyze various concrete reforms of social security.[43] Borgmann et al. (2001) provide evidence of how the reform steps taken within the time span of 1998 through 2001 have affected the sustainability of the public sector. The results of this study are shown in Fig. 6.7. The results constitute a report card for the government's policy with respect to social security. As can be seen, the sustainability gap or "true debt" was initially increased due to the suspension of the RRG99. However, further reforms legislated in the following two years led to an overall reduction of the true debt during the first term of office of the Red-Green coalition.[44]

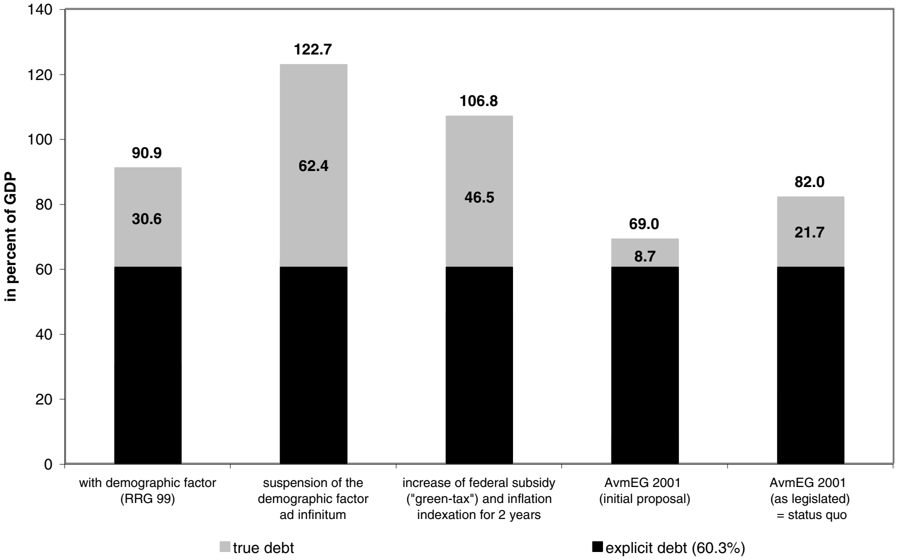
of the ratio of entitled retirees to contribution payers. This ratio is a participation-adjusted version of the old-age dependency ratio.

[41] Because of data limitations, the analysis is usually carried out with a forward-looking perspective only. Therefore the present value of life-cycle net taxes is only captured for the cohort born in the chosen baseyear.

[42] For simplification, the wealth of the public pension scheme is set to zero.

[43] These studies do not isolate the public pension scheme. Because these studies compare between different scenarios of social security they can nevertheless isolate the effect of social security **reform**.

[44] In addition, this study also quantifies the distribution of net gains and losses over the different cohorts. Making the age-specific burdens of reforms visible has been one of the most important application of generational accounting in the past years.



Notes: Baseyear 1998, entire public sector; r=3%, g=1%; 9^{th} coordinated population projection.

Source: Borgmann et al. (2001), modified for g=1%.

Fig. 6.7. The effects of social security reforms on sustainability between 1998 through 2001

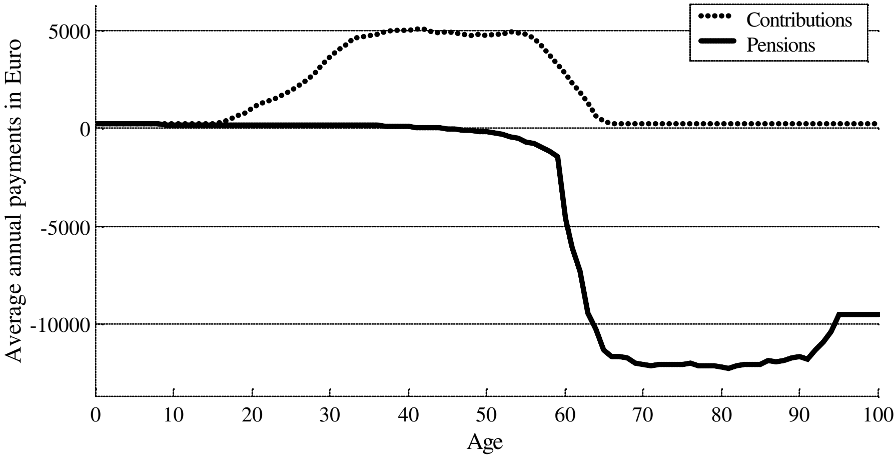
While generational accounting usually includes all taxes paid to and transfers received from the public sector, we will limit the analysis to the contributions to and the pensions that are received from the public pension scheme.[45]

The age-profiles are obtained from the German Consumer and Expenditure Survey, henceforth referred to as EVS 1998, conducted by the Federal Statistical Office in 1998 and VDR (2001) for contributions and pensions, respectively. The age-specific average payments to and from the *GRV* for a West German male are given in Fig. 6.8. The values are annual payments in Euro. The profiles obtained from panel data have been rescaled such that multiplying the profiles with the population-pyramid will exactly equal the aggregate payment streams of the baseyear. The budget of *GRV* for the baseyear 2000 is given in Table 6.8.[46]

Because the German public pension scheme is heavily financed by tax revenues from the government, it is difficult to conduct an isolated analysis of social security.

[45] We also take account of the contributions received by the public pension scheme from the unemployment insurance and of the contributions paid by the pension scheme to health insurance and long-term health insurance.

[46] Data according to national account statistics. See also Krimmer and Raffelhüschen (2003) for more details and a recent analysis of the entire public sector with the help of generational accounting.



Source: EVS 1998 and VDR (2001), author's calculations.

Fig. 6.8. Age-profile of *GRV*-contributions and benefits, West German males

Table 6.8. Receipts and expenditures of the *GRV* in 2000 (in billions)

Receipts		Expenditures	
Contr. of the employed	141.8 €	Pensions	191.6 €
Contr. of the unemployed	8.2 €	Contr. to health insurance	14.0 €
Federal subsidy	57.4 €	Contr. to long-term health insurance	1.8 €
Total	207.4€	Total	207.4 €

Source: Fetzer et al. (2003).

The federal subsidy makes up roughly 28 percent of all expenditures in the baseyear 2000.[47] Assumptions of how this federal subsidy will evolve over time will have an impact on the size of the sustainability gap. We counter this problem by looking at three scenarios: A) the fraction of expenditures that is covered by the federal subsidy will remain constant at the level prevailing in 2000, B) the federal subsidy is a constant fraction of contribution payments, and finally C) we assume that the federal subsidy in the year 2000 was completely financed by a value added tax with a tax rate of roughly seven percent and that this tax rate will be held constant.

[47] We assume that the federal subsidy is equal to the difference between receipts and expenditures as given in Table 6.8. The true federal subsidy was slightly less. Note that only pension payments, i.e. age-specific expendiutres, are considered in Table 6.8. Administration costs are not included.

6.5.2 Soft Transition – An Alternative Indicator of Sustainablility

Apart from computing the sustainability gap, it is common in the generational accounting literature to calculate indicators that show how taxes or transfers have to be adjusted in order to meet the intertemporal budget constraint. These adjustments concern either all generations or just future generations. In both cases, all adjustments are assumed to be taken immediately. Here, we develop a new indicator that is more in line with the lesson we have learned from looking at the history of the benefit rule above: reductions to the generosity of the benefit rule have been gradual but continuous. We implement this indicator of a "soft transition" by asking the following question: if benefits were reduced by one percent every year, how many times would this reduction have to occur in order to guarantee sustainability? More specifically, two different scenarios are considered: under the first (columns 4 and 5 in Table 6.9), benefits are always just 99 percent of the level of the previous year, i.e. in addition to the growth adjustment pensions are multiplied by $0.99^{year-2000}$.[48] Under the second scenario, benefits are reduced annually by one percent of the level that prevailed in 2000, i.e. the adjustment factor is equal to $1 - \frac{year-2000}{100}$ (columns 6 and 7 in Table 6.9).

For the application of generational accounting to the public pension program, the "soft transition" indicator has two advantages over the commonly used indicators of sustainability but also one disadvantage. The first advantage of the "soft transition" scenario is that it seems to be more compatible with the real life experience of political changes. Especially for social security reform, the results of Sections 6.2 and 6.4 have shown that reductions are legislated on a frequent basis aiming at short- to medium-term workability of the system. While this indicates that politicians do not seem to be completely myopic, legislation is by no means geared towards fixing long-term sustainability with one drastic measure. One can therefore interpret the "soft transition" scenario as an indicator of sustainablility that gives a rough idea of how long a transition period under a "political feasible" reform will have to be, without having to specify a concrete reform proposal.

Secondly, as will be reported in Table 6.10, this newly proposed indicator has the advantage that the results are less prone to changes of parameter values. Usually the choice of the growth rate (g) and the interest rate (r) can have severe effects on some indicators of sustainability, because the difference between r and g determines the relative importance of cash-flows that will occur in the far future. Under the "soft transition" indicator, the primary surpluses that occur at dates after the adjustment of benefits is completed tend to be near zero. Therefore the relative weight of these cash-flows is not very important for the indicator in the first place and thus the difference between r and g is not as important as it is under the other indicators. However, one important disadvantage of this indicator has to be mentioned. Because of the slow adjustment, every generation alive during the transition period is treated differently.

[48] In this scenario, we assume a real growth rate of one percent. This means that pension are constant in real terms. However, because this adjustment occurs in addition to all other already legislated reforms, a real reduction of benefits cannot be ruled out completely.

Table 6.9. Generational accounting for the German public pension scheme

Baseyear 2000; $r = 3\%$; $g = 1\%$; 9^{th} coordinated population projection

Scenarios for federal subsidy	Sustaina-bility gap (% of GDP)	Reducing benefits				
		Immediate reduction	Soft transition			
			0.99^{years}		$1 - \frac{years}{100}$	
			Years	Reduction	Years	Reduction
A)[a]	96%	18%	42	34%	33	33%
B)[b]	133%	24%	42	34%	33	33%
C)[c]	110%	20%	30	26%	25	25%

[a]federal subsidy is a constant fraction of benefit payments

[b]federal subsidy is a constant fraction of contributions

[c]federal subsidy is financed via a constant value added tax rate

Therefore, the indicator is not well suited to measure intergenerational redistribution. The traditional indicator that reduces transfers equally for all generations has a much more transparent distribution of the burdens that will arise from achieving sustainability. However, even though this equal treatment of generations is much more transparent, it also does not have a welfare economic foundation.

6.5.3 Quantifying Sustainability Under Status Quo: Results

The results of the sustainability analysis of social security are given in Table 6.9. The findings confirm that the *GRV* is currently not sustainable. Depending on the assumption concerning the future size of the federal subsidy, the sustainability gap ranges between 96 and 133 percent of GDP under current law. The sustainability gap is smaller under scenario A) than it is under scenario B) because under scenario A), the absolute size of the federal subsidy increases with the process of aging. In contrast, under scenario B), the federal subsidy behaves as the contribution receipts do. These tend to decrease because of the future decline in contribution payers. Scenario C) is more or less neutral with respect to the demographic development.

Making the *GRV* sustainable at the current contribution rate would call for an immediate decrease in benefits for all generations in the region between 18 and 24 percent, again depending on the choice of scenario. Because the immediate reduction is not a realistic indicator, we also compute a “political feasible” path into sustainability. Under the “soft transition”-indicator benefits are decreased annually by one percent for all generations from the year 2001 onwards until the intertemporal budget constraint of the public pension program is fulfilled with equality. The two versions of the “soft transition” are given in columns 4 through 7 in Table 6.9. Columns 4 and 6 show the respective duration of the adjustment periods. Columns 5 and 7 show the percentage reduction from the current status quo. As can be seen from columns 5 and 7 in Table 6.9, the percentage reduction under both scenarios is nearly the same. It is

Table 6.10. Sensitivity results for the sustainability analysis of the *GRV*

Scenario C)	Sustainability gap (in % of GDP)			Reducing benefits: Immediately (reduction in %)			Reducing benefits: After soft transition (reduction in %), 0.99^{years}			Reducing benefits: After soft transition (reduction in %), $1-\frac{years}{100}$		
	g in %			g in %			g in %			g in %		
	1.5	1	0.5	1.5	1	0.5	1.5	1	0.5	1.5	1	0.5
$r = 2\%$	389	220	149	23.1	22.7	21.5	25.3	26.0	26.0	25	26	26
$r = 3\%$	151	110	83	21.6	20.2	18.8	26.0	26.0	25.3	26	25	24
$r = 4\%$	84	66	53	18.9	17.6	16.3	25.3	24.5	23.0	25	24	22

only the time span that is needed to reach this reduction that differs by quite a bit between these two different transition scenarios. Finally, we can see that there is quite a difference between the immediate reduction of benefits and the soft transition. Under scenario C), the slow reduction calls for a benefit reduction by 26 percent instead of 20 percent under the immediate reduction. The necessary reductions are even larger under scenarios A) and B), amounting to over 34 percent.[49]

Results of a sensitivity analysis for different values of r and g are reported in Table 6.10. For the sake of a clearer presentation, only one scenario for the development of the federal subsidy is reported. Scenario C), where the federal subsidy is financed via a value added tax, is chosen because it is the scenario under which the newly proposed indicator is the most sensitive to changes of the parameter values. As can be seen from Table 6.10, the "soft transition" indicator is still much less sensitive to changes of parameter values than are the other currently used indicators. Specifically, the difference between the results for the chosen parameter values are at the maximum, four percentage points under the "soft transition" indicator, whereas the maximum difference for the indicator where all transfers are reduced immediately amounts to 6.8 percentage points.

Summarizing, we have shown that under a politically realistic transition path a further continuous decrease of benefits that will, at the end of the transition, amount to a reduction of something between 26 and 34 percent is necessary to make the German public pension program sustainable. While the results from the generational accounting perspective are not really comparable to our measure of relative generosity, it seems that the reductions of the past 25 years – including the reform of 2001 – have taken us about halfway. Roughly, another 30 to 40 years of continuous bite-size reductions will be necessary to cope with the severe demographic transition Germany is undergoing right now.

[49]Note that under scenario A), the federal subsidy is also decreased with the reduction of benefits while under scenarios B) and C), the federal subsidy remains constant.

6.6 Summary

In this chapter, we analyze the political volatility of social security wealth by constructing a measure of relative generosity of the benefit rule of the German public pension system. In order to do so, we first need to outline the social security reforms of the past 40 years. We have tried to put this chronicle of the benefit rule into an economic structure, and as a result, we come up with a history of the benefit formula of the German public pension scheme from an economist's point of view. We believe this to be a nice by-product of our analysis, since a comparable history of the German *GRV* can only be found in fragments.

One of the two main issues we address is the uncertainty, or "political risk", of benefit payments. We show that changes to the generosity are not only very frequent but are also of significant magnitude. When stressing the risk-aspects of alternative forms of old-age income provision, such as real investments, one should keep in mind that the pay-as-you-go financed pension system bears several significant risks itself. We have only pointed out one of those risks here, namely the "political risks".

As the second main focus, we have tried to estimate a policy function that tries to explain legislative changes of the benefit rule by current and future demographic developments. Our findings are that a 17 year forward of the old-age dependency ratio has a significant negative influence on the pension scheme's generosity to both the middle aged and the elderly. Apparently, legislation steers towards medium-term sustainability of the system, and it is not blind to demographic development. Even though the demographic factor that was intended to be a part of the benefit formula under the RRG99 was never implemented, it seems as if German pensions have been subject to an implicit demographic factor for quite some time.

Furthermore, both Sect. 6.2 and 6.4 do not show evidence that elderly people are protected from social security reform. On the contrary, the generosity for the elderly is also influenced by the future development of the old-age dependency ratio. This indicates that the burden of guaranteeing the medium-term sustainability of the system is also carried by the currently old. We believe this phenomenon to be a point of departure for future models of political economy of social security.

Finally, we derive the magnitude of future changes that are necessary in order to guarantee the long-term sustainability of the *GRV*. If the generosity is only reduced by annual bite-size steps, as it has been the case in the past, reductions will amount to roughly 30 percent of the current level. Instead, if measures were taken immediately, a reduction of only circa 20 percent would be required.

Chapter 7
Social Security, Portfolio Choice, and Financial Markets

Asset allocation for old-age provision has several features that makes it unique from a standard (short term) investment decision. First, the investment horizon is unusually long and changes with age. Second, social security wealth and human wealth must be considered as a "quasi-asset" next to financial wealth. Both of these subjects will be covered in the next Sect. 7.1 dealing with social security and portfolio choices. Also, we show how the government should take account of the advantages of risk diversification via PAYG. A portfolio approach to designing social security is presented. In Sect. 7.2, we look at social security in the light of financial products. More specifically, we touch on the issue of annuities as instruments in old-age provision, and we show how financial markets can be used to price the cost of minimum guarantees. To conclude, we point to the future by showing how new – currently non existent – capital markets and financial products could help in managing risk more efficiently.

7.1 Social Security and Optimal Portfolio Diversification

When looking at the portfolio composition for old-age provision, it would be foolish to neglect that social security wealth plays an important role as a "quasi-asset" in the portfolio of an individual's wealth. Taking this into consideration should have an effect on personal investment decision when no risk-free asset is available (Sect. 7.1.1). At the same time, on the bases of welfare-improving risk diversification, the policy maker should also take a portfolio approach when designing social security (Sect. 7.1.2). In the same line of argument, international diversification of the portfolio should be recognized as another possible way of spreading risk. Because domestic capital markets and labor productivity – and hence the return of social security – are positively correlated at long horizons, international diversification is of special importance for old-age provision (Sect. 7.1.3). Finally, in Sect. 7.1.4, we point out that optimal portfolio decisions may change over the life-cycle. An individual close to retirement will most likely prefer a different portfolio composition than an individual at the beginning of her working life. There a two reasons that cause this portfolio pattern over the life-cycle. First, the investment horizon changes with the remaining life-expectancy and second, the alternative risk-factors, i.e. the size and composition of risk coming from non-financial wealth (e.g. human capital), vary depending on the phase in life.

Some important caveats should be mentioned before we proceed with constructing efficient portfolios. As is always put forward as a critique of the CAPM (Capital Asset Pricing Model), we use historical data to evaluate the composition of risk and return for the future. This is especially critical in our case, since – as has been shown in the previous sections – the demographic changes may have an important impact on the development of the future returns of the different assets. These effects are of course not captured in the historical data. Secondly, in a similar line of argument, the return and standard deviation reported for social security below does not take account of the large part of the "political risk" underlying the return to social security. That this risk is non-negligible was derived for the case of Germany above. This point is of particular relevance, since many of the political changes are already legislated. Finally, we employ the static version of the CAPM in the simple mean-standard deviation diagram throughout most of this Section.[1] Nevertheless, we believe the analysis conducted below to be instructive in illustrating the risk diversification features and benefits of social security. The derived results should be viewed upon from this perspective and not be taken as investment advice.

7.1.1 Portfolio Choice in the Presence of Social Security

In order to motivate the risk diversification aspects of social security, we look at optimal portfolio choices when social security can be seen as a tradable financial asset. In order to do so, we utilize the analysis of risk and return of portfolios as pioneered by Markowitz (1959). We draw on the empirical work of Baxter and King (2001) and Schacht (2001) for the United States and Germany, respectively. Specifically, we look at the efficient portfolio frontier in a mean–variance set–up with and without social security. In a first step, we look at the efficiency frontier when only equity and bonds are available. As a next step, we introduce social security as a "normal" capital good, i.e. as a tradable asset for which short sales are also possible.[2] In a third step, we take the weight of social security in the portfolio as exogenously given. This exercise serves two purposes. First, we want to point out that social security fills a missing market, since apparently the risk-return structure cannot be duplicated by a combination of other assets.[3] Second, we want to point out that social security is an important element in old-age provision. Neglecting its existence when thinking about the portfolio allocation of retirement savings would be foolish.[4]

The analytical derivation of the efficient portfolio frontier is well known from the work of Merton (1972). The basic concept is to find the portfolio allocation, i.e. the weights of the single assets in the portfolio, that minimize the variance of the portfolio under the constraints that the expected portfolio-return is equal to a given value and that the sum of the weights equals one. The efficiency frontier is then constructed by performing this calculation for a grid of values for the portfolio return. Table 7.1 summarizes the necessary statistics for the calculation of the efficiency frontier for

[1] We will comment on expansions in Sect. 7.1.4.

[2] In the next section, we are using a portfolio approach for the design of social security. This means that we are calculating the optimal size of social security in the portfolio under assumptions of utility and risk aversion when individuals will optimize their financial portfolio,

Table 7.1. Data for the calculation of the efficient portfolio frontier, USA and Germany

Panel A: Case Study United States

Social Security and Financial Assets (annual data, 1970–1997)

		Social Security	Equities			Long-Term Gov. Bonds		
			United States	Europe	Far East	USA	Europe	Asia
Mean return (% per year)		0.24	8.43	9.12	11.2	5.04	5.90	5.91
Standard deviation (% per year)		2.16	16.17	18.8	29.68	11.84	13.21	14.74
			Correlations with Other Assets					
Social Security		1.00	0.33	0.23	0.40	0.18	0.19	0.29
Equities	USA	0.33	1.00	0.68	0.27	0.58	0.21	0.16
	Europe	0.23	0.68	1.00	0.48	0.42	0.58	0.43
	Far East	0.40	0.27	0.48	1.00	0.13	0.34	0.60
Bonds	USA	0.18	0.58	0.42	0.13	1.00	0.42	0.23
	Europe	0.19	0.21	0.58	0.34	0.42	1.00	0.75
	Asia	0.29	0.16	0.43	0.60	0.23	0.75	1.00

Panel B: Case Study Germany

Social Security and Financial Assets (annual data, 1976–1998)

	Social Security	German Assets			Foreign Assets	
		Stocks (DAX)	Bonds (REX)	Real Estate	S&P500	Bonds
Mean return (% per year)	4.30	12.40	8.10	3.30	19.60	9.30
Standard deviation (% per year)	1.73	23.37	5.48	5.10	19.67	9.90
		Correlations with Other Assets				
Social Security	1.00	-0.45	0.00	0.57	-0.09	0.12
Stocks (DAX)	-0.45	1.00	0.20	-0.10	0.44	0.47
Bonds (REX)	0.00	0.20	1.00	-0.14	0.26	0.52
Real Estate	0.57	-0.10	-0.14	1.00	0.07	0.12
S&P500	-0.09	0.44	0.26	0.07	1.00	0.72
Foreign Bonds	0.12	0.47	0.52	0.12	0.72	1.00

Notes: The statistics for the United States are not comparable with those for Germany: for the U.S., all returns are in real terms, whereas for Germany, the statistics are based on nominal returns.
Source: Baxter and King (2001) and Schacht (2001).

the United States (Panel A) and Germany (Panel B). The data are taken from and Baxter and King (2001) and Schacht (2001)[5], respectively. Detailed descriptions of the respective data can be found there. Also, we include the statistics necessary for the construction of the portfolios with international assets (U.S. and Germany) and real estate (Germany only). These statistics will only be put to use in the analysis of Sect. 7.1.3.

The numbers for the two countries given in Table 7.1 are not comparable with each other, since the data for Germany is nominal. As a result, the returns for all assets in Germany are quite large. The German case study has to be viewed with some reservations for this lack of adjustment to real values.[6] An interesting aspect of the German case study is the inclusion of real estate wealth. We will show later that including real estate in the portfolio will reduce the difference between the efficiency frontier for the cases with or without social security. The original study for Germany also includes the money market. We have not taken this asset into the presentation here. Because of the lack of a safe asset over long horizons, the mutual fund theorem of Tobin (1958), which states that all investors should hold the same mix of stocks and bonds, does not apply.[7]

In Fig. 7.1, we depict the efficient portfolio frontier for both countries in the "stripped down" case: we only consider domestic equity, bonds, and social security. We depart from a situation where only equity and bonds are part of the portfolio (grey line) and than add social security as a tradable asset (black line). In Table 7.2, we also

taking the social security component as given. In this analysis, we will not consider short sales.

[3]However, with the German data, one can show that just using equity and bonds overstates the case in favor of social security. See Sect. 7.1.3, where we include real estate in the portfolio for Germany.

[4]Poterba et al. (2003) use a much more sophisticated model to analyze the utility evaluation of risk in retirement savings accounts in the United States, the so-called 401(k) plans. They compare expected utility for alternative investment strategies for the 401(k) plan from the household level by calibrating simulations to the lifetime profiles of social security earnings records from the Health and Retirement Survey for the United States. Their conclusion is not so different from our line of argument here: "We find that the expected utility of retirement wealth is very sensitive to the value of wealth held outside the defined contribution plan, including both liquid wealth and annuitized wealth such as prospective social security benefits or defined benefit plan payouts." (Poterba et al. (2003, p.2))

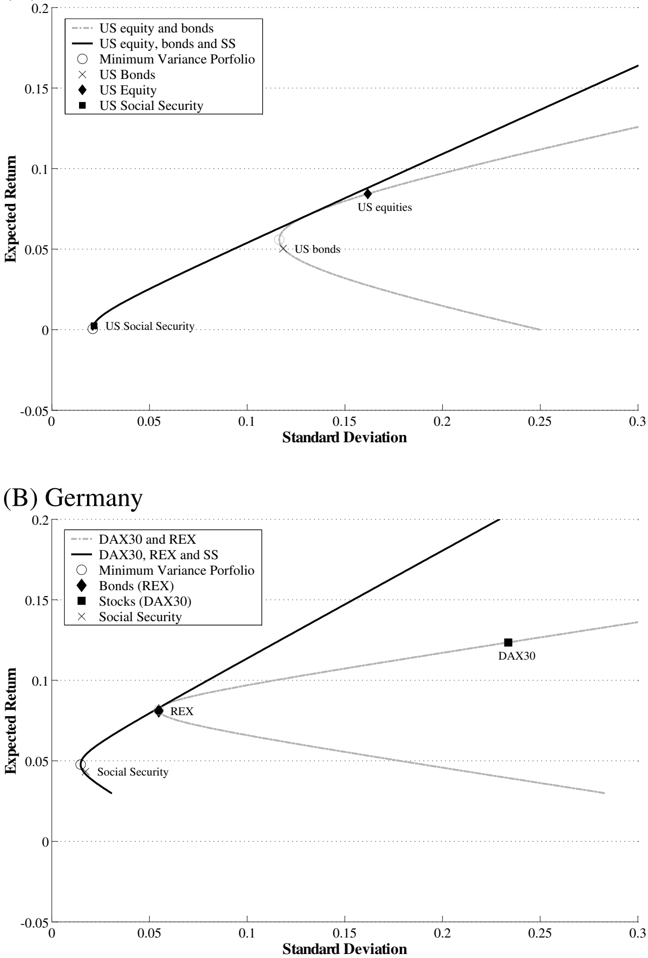
[5]Note that in Schacht's (2001) data, there are inconsistencies between the variance-covariance matrix in the appendix (p. XXXVIII) and the coefficients of correlation given in the text (p.59). We have used the data according to the variance-covariance matrix.

[6]The validity of the cited data for the return of the U.S Social Security has also been put into question. Cf. discussion of Baxter and King (2001) in Campbell and Feldstein (2001, 434–436).

[7]It is sometimes argued that long-term bonds should be viewed as a safe asset for long-term investors, even though bond returns are not safe for the short-term investor. Cash (or money market) are, on the other hand, interpreted as a safe asset in the short-term but can not be viewed as safe for the long-term investor. See Campbell and Viceira (2002, pp.86,87).

(A) United States

(B) Germany



Source: Baxter and King (2001) and Schacht (2001)

Fig. 7.1. Efficiency frontier: USA & Germany with bonds, equity, and social security

report the composition of different efficient portfolios for both cases. The results show that social security helps to "stretch" the efficiency frontier considerably to the "left". The minimum variance portfolios in the presence of social security feature much less risk than those without social security. This is due to the low standard deviation of social security in both country studies and the non-existence of a truly safe assets.

Accordingly, the asset social security dominates the other assets in the minimum variance portfolio with a portfolio weight of 101 (92) percent in the United Stated (in Germany). In order to achieve a portfolio return of five percent in the Untied States, 40 percent are invested in equity and the remaining 60 percent are split nearly

Table 7.2. Composition of efficient portfolios for the cases with and without social security

Panel A: United States

Without Social Security				With Social Security				
Expect. Return	*Stand. Dev.*	Portfolio Weights ω^e	ω^b	*Expect. Return*	*Stand. Dev.*	Portfolio Weights ω^{ss}	ω^e	ω^b
Minimum Variance Portfolio								
5.6	*11.6*	16.2	83.8	*0.05*	*2.1*	101.0	-4.1	3.0
Efficient Portfolio for a Return of ...								
5	*11.9*	-1.2	101.2	*5*	*9.3*	30.2	41.5	28.3
10	*21.0*	146.3	-46.3	*10*	*18.3*	-41.5	87.6	53.9
15	*39.0*	293.8	-193.8	*15*	*27.5*	-113.1	133.7	79.5

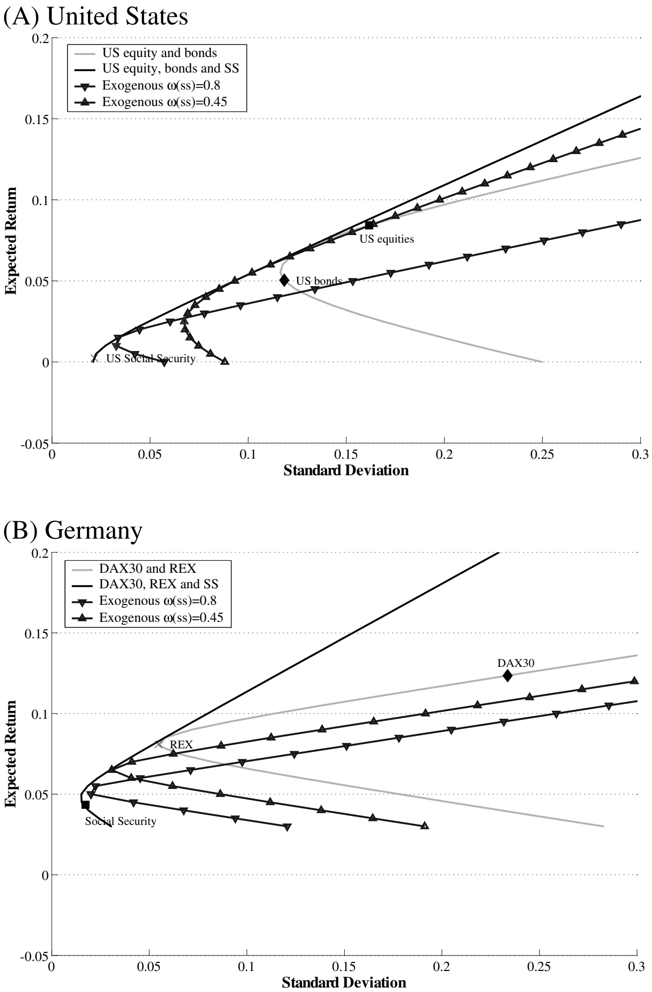
Panel B: Germany

Without Social Security				With Social Security				
Expect. Return	*Stand. Dev.*	Portfolio Weights ω^e	ω^b	*Expect. Return*	*Stand. Dev.*	Portfolio Weights ω^{ss}	ω^e	ω^b
Minimum Variance Portfolio								
8.1	*5.5*	1.0	99.0	*4.8*	*1.5*	92.2	3.2	4.6
Efficient Portfolio for a Return of ...								
5	–	–	–	*5*	*1.5*	86.4	3.5	10.1
10	*11.4*	44.7	55.3	*10*	*8.0*	-39.8	9.5	130.3
15	*37.4*	162.4	-62.4	*15*	*15.4*	-166.0	15.5	250.5

Notes: All numbers are in percent. ω^{ss}, ω^e and ω^b denote the portfolio weights of social security, equity, and bonds, respectively.
Source: Author's calculation using data from Baxter and King (2001) and Schacht (2001).

equally between bonds and social security. To generate large expected returns, the efficient strategy would be to short social security and invest in equity and bonds, with a larger share in equity than bonds. In Germany, equity is not a very attractive investment. Because of the high risk associated with equity, this asset does not play an important role in the portfolio in the case with social security. The investor will rather short social security to finance large purchases of bonds in order to achieve large expected returns. Summarizing, we see that social security is an important addition to the portfolio: the asset social security permits the possibility of a much lower risk exposition. For aggressive investors, social security would also be valuable as a asset that can be shorted in order to finance high return investments.

The assumption of social security being a normal capital good that is tradable in any amount (including short sales) is not very realistic. We therefore compute the efficiency frontier for exogenous shares of social security. According to OECD (2001), social security transfers make up roughly 80 percent of non-labor income



Source: Author's calculation using data from Baxter and King (2001) and Schacht (2001)

Fig. 7.2. Efficiency frontier with an exogenous share of social security wealth

of retirees in the age between 65 and 74 in Germany. The respective value in the United States is much lower at about 45 percent on average. In Fig. 7.2, we augment the results from Fig. 7.1 by the efficient portfolio frontier when the social security weight of the portfolio is exogenously given at 45 and 80 percent, respectively. Forcing individuals to invest 80 percent of their old-age provision into social security provides the possibility of realizing portfolios with low level of risk. However, higher returns are only achievable when individuals are exposed to considerable more risk than they would be if no contraction to social security at such a high level would exist. This can be verified by checking that the 80 percent social security efficient portfolio frontier is beneath the efficiency frontier without social security for returns equal to or above the return on bonds. This holds for both Germany and the United States. The reason for this is that because the individual's wealth is already tight to social security, the investor will need to short bonds and heavily invest into equity in order to reach larger returns. This is associated with a higher level of risk.

For the case with 45 percent of wealth forced into social security investment, the minimum variance portfolio bears more risk than in the 80 percent case. However, higher returns are feasible at a lower exposure to risk. For the United States, one can see that the 45 percent case dominates the efficiency frontier without social security. For Germany this is still not the case: if the investor is looking for an expected return that is larger than the return of bonds, she would be better off without social security. So we see that if social security is an assets for which the portfolio weight is exogenously fixed, the existence of social security does not necessarily improve the situation of the investor. Social security will only be valuable to the investor if the risk aversion of individuals is sufficiently high, such that individuals will always prefer to invest in a portfolio with a lower standard deviation than that of the pure bond-portfolio. If this is not the case, social security is only helpful when the government chooses the correct level of provision from social security. The optimal policy will depend on the utility of individuals and the covariance matrix of the assets. In Sect. 7.1.2, we derive how an optimal policy rule will look like.

7.1.2 A Portfolio Approach to Designing Social Security

Following Matsen and Thøgersen (2004), we construct a model to show how the portfolio approach can be used in designing optimal social security. We concentrate on the case with perfect capital markets, where it is not welfare improving for the public pension program to hold risky assets in a trust fund.[8]

Consider a two period OLG-model, similar to that presented in Chap. 5, with a stochastic demographic growth rate $\hat{\eta}$ and stochastic wage growth rate $\hat{\epsilon}$. Note that the stochastic process for the wage-rate differs here from that in Chap. 5, since now the wage growth rate is stochastic and not the wage rate itself. With this specification, the log of the wage rate follows a random walk.[9] We thus have:

$$N_{t+1} = (1 + \hat{\eta}_{t+1})N_t \tag{7.1}$$

$$w_{t+1} = (1 + \hat{\epsilon}_{t+1})w_t \tag{7.2}$$

Further, define the aggregate growth rate to be $\hat{g}_{t+1} \equiv \hat{\eta}_{t+1} + \hat{\epsilon}_{t+1} + \hat{\eta}_{t+1}\hat{\epsilon}_{t+1}$. Savings can be allocated to a risk free bond with a real rate of return of r^f, or to a risky asset with return $\hat{r}_{t+1}$. The weight of the risky assets in the savings-portfolio of the individual will be called ω^p. The remaining share of savings, $1 - \omega^p$, will be allocated to the risk free bond. By assumption, all stochastic variables are distributed lognormal. The public pension program collects τ percent of the labor income from the

[8]See discussion at the end of this section.

[9]The specification of the process of stochastic labor income is important for the results of risk sharing within social security schemes; cf. Thøgersen (2003).

young and pays out a benefit β to every member of the old generation. The adjustment mechanism is of the "wage-indexed defined contribution" type, such that the pension policy is described by: $\beta_{t+1} = \tau w_t(1 + \hat{g}_{t+1})$. For simplicity, the households are assumed to consume only in the second period of life and utility features constant relative risk aversion (CRRA):

$$U_t = \mathrm{E}_t \left[\frac{(c^o_{t+1})^{1-\varsigma}}{1-\varsigma} \right], \tag{7.3}$$

where ς is the coefficient of relative risk aversion, which is constant across generations. From the assumptions taken above, one can derive second period consumption to be:

$$\begin{aligned} c^o_{t+1} &= w_t(1 + \hat{r}^p_{t+1}), \text{ with} \\ \hat{r}^p_{t+1} &\equiv r^f + \omega^p(1-\tau)(\hat{r}_{t+1} - r^f) + \tau(\hat{g}_{t+1} - r^f) \end{aligned} \tag{7.4}$$

The model can be solved by applying a Taylor approximation on $\hat{r}^p_{t+1}$ and making use of the fact that, under the assumptions of log-normality, maximizing Eq. (7.3) is equivalent to maximizing $E_t[\log U_t] = E_t[\log(1+\hat{r}^p_{t+1}) - \log(1+r^f)] + \frac{1}{2}(1-\varsigma)\sigma^2_p$, where σ^2_p is the variance of $\log(1+r^p_{t+1})$; see Campbell and Viceira (2002, pp.20–27) and Matsen and Thøgersen (2004, p.7).

Solving for the optimal share of risky assets of total financial assets for a given contribution rate τ yields:[10]

$$(\omega^p)^* = \frac{\mu^r + \frac{1}{2}\sigma^2_r}{(1-\tau)\varsigma\sigma^2_r} - \frac{\tau}{1-\tau} \cdot \frac{\sigma_{rg}}{\sigma^2_r}, \tag{7.5}$$

where we define μ^z as the expected excess-return over the safe bond: $\mu^z \equiv \mathrm{E}\big[\log[1+z] - \log[1+r^f]\big]$ for $z = \{\hat{r}_{t+1}, \hat{g}_{t+1}\}$. σ_z is defined as the variance of $\log(1+z)$ and σ_{zx} is the covariance of $\log(1+z)$ and $\log(1+x)$. The optimal share of risky assets in financial assets given in (7.5) is determined by the excess return of assets over bonds, the variance of r, the covariance of r and g, the degree of risk aversion (ς) and the contribution rate to social security (τ). For obvious reasons, the optimal value of ω^p increases with the excess return of assets, but decreases with its variance and with higher values of risk-aversion. The effect of the social security contribution rate on $(\omega^p)^*$ is ambiguous depending on the sign and size of the covariance of r and g.

However, this ambiguity can be transformed into a clear dependency on the sign of σ_{rg} when we look at the portfolio-weights in terms of total wealth instead of just

[10] We only consider the case of "traditional risk sharing" here. This type of risk sharing is similar to *ex-post* risk sharing. Matsen and Thøgersen (2004) also consider what they call "Rawlsian risk sharing". Even though this type of risk is closer to the *ex-ante* perspective taken in Chap. 5, the contribution rate τ^*_{Rawls} is nevertheless lower than τ^*_{trad} for realistic parameters of relative risk aversion ($\varsigma > 1$). This contradicts the discussion in 5.5.2. It can be explained, however, by the rather extreme specification of the stochastic process for the wage rate.

financial wealth. Total wealth is divided into a share of τ percent "invested" in social security wealth (ω^{ss}), the fraction $(1-\tau)\omega^p$ percent of total wealth is allocated to the risky asset (ω^e), and the remaining $(1-\tau)(1-\omega^p)$ percent make up the share of total wealth invested in the risk-less bond (ω^b). So when looking at the fraction of risky assets in total wealth we have:

$$(1-\tau)(\omega^p)^* = \frac{\mu^r + \frac{1}{2}\sigma_r^2}{\varsigma\sigma_r^2} - \tau \cdot \frac{\sigma_{rg}}{\sigma_r^2}. \tag{7.6}$$

It is clear to see that the share of total wealth invested in risky assets will only be positively dependent on τ when social security can serve as a hedging instrument, i.e. when the covariance of r and g is negative. On the other hand, if the return from assets and social security are positively correlated, the fraction of assets in total wealth will always decrease with τ. The first term on the RHS of Eq. (7.6) is equal to the optimal share of risky investments for the case without social security. Note that the chosen specification of utility has some very specific assumptions on the influence of background risk[11] on the share of risky investments: namely, it has none. Only in such a setting does τ not have an influence on the size of the risky portfolio, i.e. is equal to the case without social security when the correlation between r and g is zero even though the variance of g is positive: $\frac{\partial(1-\tau)(\omega^p)^*}{\partial\tau}\big|_{\sigma_{rg}=0,\sigma_g^2>0} = 0$ with $\tau > 0$ and $\sigma_g^2 > 0$. However, the existence of a risk-free asset is also a strong assumption. Empirical evidence presented in the section above suggests that the variance of social security returns is less than that of bonds. So that commonly, one would expect social security to reduce background risk instead of increasing it, as is the case here.

The optimal contribution rate to social security may now be derived from maximizing utility and taking into consideration that individuals will adjust their optimal portfolio depending on τ. The optimal contribution rate can be calculated to be:

$$\tau^* = \frac{\left(\mu^g + \frac{1}{2}\sigma_g^2\right) - \left(\mu^r + \frac{1}{2}\sigma_r^2\right) \cdot \frac{\sigma_{rg}}{\sigma_r^2}}{\varsigma\left[\sigma_g^2(1-\rho_{rg}^2)\right]} \tag{7.7}$$

where $\rho_{rg} \in [-1, 1]$ is the correlation coefficient between equity returns and the return from the PAYG social security. The term in squared brackets in the denominator of (7.7) is the systematic risk of the PAYG social security. It is systematic because the social security risk cannot be hedged against with other assets. This risk is particularly large if the social security risk is statistically independent from other risks, i.e. if ρ_{rg} is near zero. Since a riskless asset exists by assumption and social security is risky in the present model, the size of τ^* is negatively dependent on the degree or relative risk aversion. The denominator therefore expresses how much risk is added to the total wealth portfolio and how this increased risk is evaluated by individuals.

On the other hand, the optimal contribution rate is an increasing function in the expected excess return of social security over the risk-free bond.[12] The sign and

[11] See Eichner and Wagener (2004) on utility assumptions and background risk.
[12] Note that $\mu^g + 0.5\sigma_g^2 = \log E_t\left[(1+\hat{g}_{t+1})/(1+r^f)\right]$.

magnitude of the second term in the numerator of Eq. (7.7) is determined by the covariance of r and g. If the covariance is negative, the expected excess return of both PAYG social security and the risky asset can be exploited, while the risk can be partially "hedged away" by the two assets. In this case, the optimal share of the portfolio in the risk-free asset, $(\omega^b)^* = (1 - \tau^*)(1 - (\omega^p)^*)$, is unambiguously decreasing.

Also, from Eq. (7.7) one can see that a necessary condition for a positive contribution rate is either a positive excess return of social security or a negative correlation between g and r. That is, either the investment in social security has a "money's worth" by itself (excess return) or it serves as a unique device for hedging asset risks (negative correlation). An alternative interpretation of this result has been stressed by Hauenschild (1999): social security can be welfare improving for diversification-reasons, even though its expected return is below the interest rate. Note that intergenerational risk sharing is not necessary for this result.

The discussion above was restricted to the case with perfect capital markets, in the sense that all individuals have perfect and costless access to capital markets. In such a setting, there is no reason for social security to build up a trust fund that invests in assets because such a policy results in a "shell game" (cf. Abel (2001b)).[13] The real allocation in the economy is not altered by investments of social security in equity, because private households will adjust their portfolio composition in response to the trust fund's investment in risky assets. Portfolio changes of the trust fund are completely neutralized by private households.[14] Building on the empirical evidence of Mankiw and Zeldes (1991), who find that a majority of consumers do not own stocks, Abel (2001a) argues that fixed costs may prevent some households from holding stocks. Households in the lowest income-percentiles are especially prevented from participating in the stock markets due to the fixed costs associated with investing in risky assets. Matsen and Thøgersen (2004) model an extreme case of imperfect access to capital markets: the individuals do not hold any risky assets at all, i.e. $\omega^p = 0$. In such a case, the optimal contribution rate to social security is equal to $\tau^* + (1 - \tau^*)(\omega^p)^*$, of which the trust fund will invest $(1 - \tau^*)(\omega^p)^*$ into equities. Thus, the optimal policy in the constrained case is a replication of the optimal policy in the unconstrained case.

We conclude this section with some numerical illustrations applying the results derived above.[15] In Table 7.3, we show the baseline values of the key variables and parameters and the therewith computed values of (ω^*) and τ^* for Norway, Sweden,

[13] See also Pestieau and Possen (2000).

[14] The argument is similar to the Ricardian equivalence proposition in public finance or the Modiglian-Miller theorem in corporate finance. This result will only hold if the trust fund is designed as an individual account scheme. If instead, future contribution payments are adjusted in response to good or bad equity returns, trust fund investments may have an impact; see the discussion of Bohn's (1999) result in Sect. 4.2.

[15] The following should really be understood as a numerical illustration and not as empirical evidence.

Table 7.3. Optimal portfolio choice and social security design: Numerical illustrations

	Norway	**Sweden**	**UK**	**USA**
Period	1970–1999	1920–1998	1919–1998	1891–1998
Values from historical data				
$E[\log(1+\hat{r})]$	5.07%	7.07%	7.41%	6.93%
σ_g	3.57%	6.28%	3.50%	5.77%
σ_r	34.85%	18.65%	21.69%	18.67%
ρ_{rg}	-0.298	0.055	0.087	0.112
Projected and assumed values				
$E[\log(1+\hat{g})]$	1.69%	2.20%	1.88%	2.96%
$\log(1+r^f)$	2.00%	2.00%	2.00%	2.00%
ς	5	5	5	5
Results: Optimal portfolio without social security ($\tau = 0$)				
ω^p	15.06%	39.16%	32.99%	38.28%
risk-free bonds	84.94%	60.84%	67.01%	61.72%
Results: Optimal portfolio with social security				
$(\omega^p)^*$	16.00%	45.01%	32.99%	79.68%
$\tau^* = (\omega^{ss})^*$	4.94%	13.56%	0.00%	54.32%
risky assets: $(\omega^e)^* = (1-\tau^*)(\omega^p)^*$	15.21%	38.91%	32.99%	36.40%
risk-free bonds: $(\omega^b)^*$	79.85%	47.53%	67.01%	9.28%

Notes: Results with social security are based on the assumptions of perfect capital markets and "traditional risk sharing".
Source: Matsen and Thøgersen (2004).

UK, and USA.[16] The following interesting observations emerge: for Norway and the United Kingdom, the expected excess return of social security over the risk-free bond is negative.[17] However, in Norway social security is nevertheless part of the optimal portfolio due to the negative correlation between r and g ($\rho_{rg}^{Norway} < 0$). Social security is therefore valuable as it serves as a hedging device for risky capital. On the other hand, in the United Kingdom, the necessary condition for social security to be a part of the optimal portfolio[18] is violated because the excess return is negative and the covariance of r and g is positive. Thus, the contribution rate is zero as the risk-free

[16] See Matsen and Thøgersen (2004) for a description of the data.
[17] To be precise, the excess returns are -0.246% and -0.059% for Norway and the UK, respectively.
[18] We do not allow for short sale of social security here. Otherwise the optimal portfolio would include a short position in the social security scheme. For the matter of designing social security, this seems odd. For an individual portfolio decision, it might be plausible to wish for such a tradable asset (see above).

bond is more favorable than social security both in regard to risk diversification and return. Also, note that the overall share of risky assets rises in the Norwegian case in the presence of social security in comparison to the case without social security. This is only the case for Norway and is again due to the fact that social security is a hedge for risky capital, which enables individuals to increase their exposure to this risky equity. Finally, note that the optimal contribution rate is so much larger in the USA in comparison to Sweden, because social security is a much more attractive investment in the USA by itself. This can be verified by calculating the Sharpe values for social security (μ_g/σ_g) in the respective countries: for Sweden, this ratio is equal to 0.063, while for the USA, it is 0.195 (the Sharpe values for risky capital are nearly the same for both countries at approximately 0.36).

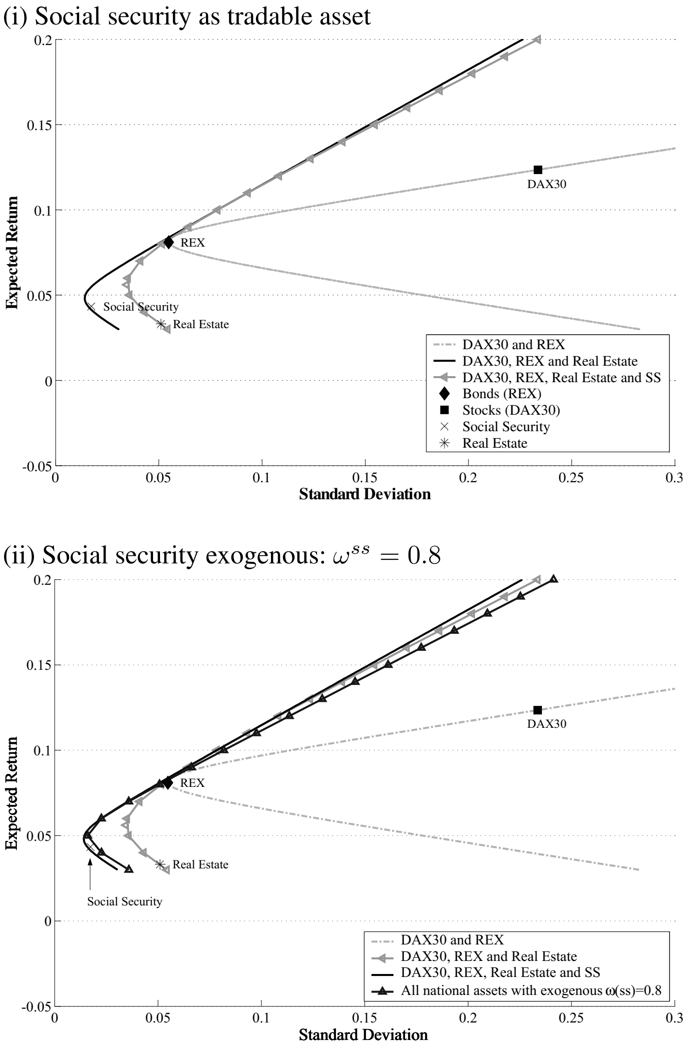
7.1.3 More Diversification: Real Estate and International Assets

Inclusion of Real Estate. In the analysis above, the available financial assets were restricted to equity and bonds. This constitutes a rather restrictive selection. The data of Schacht (2001) allows us to expand the analysis by including real estate wealth as an asset in the German portfolio (see Table 7.1). Including real estate wealth is interesting for two reasons: first, real estate is a very popular vehicle for savings for retirement. Second, as has been alluded to in Sections 2.3.3 and 5.5.2 from a theoretical point of view, land may have an important role in the intertemporal allocation of resources. Land owes this special role to three very specific features: land is durable, its supply is fixed and it yields a positive stream of dividends over time.[19]

In Fig. 7.3, we show the influence of including real estate wealth in the portfolio. We again show the comparison of the efficient portfolio frontier for the cases with and without social security when social security is a tradable asset (i) and when social security is a exogenously fixed percentage of the portfolio (ii). For the sake of a clearer presentation, we confine the second case to a given percentage of 80 percent. The figure shows that the initial "no-social-security-portfolio" allows much better risk diversification with real estate than without real estate (compare the grey line and the line with dotted triangles). Thus, the introduction of social security (solid black line) does not change the efficient portfolio frontier as much as in the case before. In particular, portfolios with higher returns than the bond-return (REX) allow for very similar optimal risk-return combinations whether social security is present or not. The main advantage of social security is still the introduction of an asset that is less risky than any other available asset. Hence, the efficiency frontier with social security generates more preferable allocations for returns lower than the bond-return.

The theoretical findings cited above imply that in terms of intertemporal allocation, real estate and social security provide similar services. Interestingly, the data taken from Schacht (2001) show that the co-movements of real estate and social security also have similarities (see Table 7.1). For the purpose of risk diversification

[19]There are of course some differences between land and real estate. But real estate is nevertheless a pretty good proxy for such an asset. For the analysis, we neglect that real estate is a less liquid asset and not perfectly dividable in many cases. However, the financial industry has been offering open property funds for quite some time.



Source: Author's calculation using data from Schacht (2001)

Fig. 7.3. Efficiency frontier, national assets with real estate, Germany

these two assets thus also exhibit similarities. As a result, it comes as no surprise that the changes to the location of the efficient portfolio frontier between the cases with or without social security is much less when real estate is part of the basket of available assets. The risk diversification benefits of social security are hence over-estimated when real estate is not taken account of as a tradable asset.

On the other hand, forcing individuals into social security (depicted in sub-figure (ii)) does not harm the risky type of investor anymore. Here we speak of "harm" in a sense that the efficient portfolio frontier is more restricted with mandatory social security than without it. Remember from Fig. 7.2 that this was the case without real estate. Instead, it becomes more easy to unambiguously improve the efficient portfolio frontier with the inclusion of social security even when ω^{ss} is exogenously given. The reason is social security is "portfolio-improving" for a low standard deviation (as it was the case without real estate), and at the same time, does not reduce expected returns for a high level of risk (standard deviation). The latter effect does not occur, because the possibility to short real estate allows the risk-loving investor to counteract the forced over-investment into social security.

International Diversification. According to some crude calculations by Shiller (1999b), the standard deviation of world change in real per capita GDP is about half that of the average taken over the within-country changes. The variance of changes in real per capita GDP is thus on average four times as large for a single country than that variance for the world as one. Shiller infers the data is consistent "with the plausible notion that international risk sharing is on the same order of magnitude importance as the within-country intergenerational risk sharing."[20]

Shiller (1999b) further argues that a solution for optimal risk sharing on an international and intergenerational level should ensure that both old and young incomes are perfectly correlated with world income. On first sight, an optimal pay-as-you-go social security scheme that implements such a solution is rather complicated: it takes account of the variance-covariance matrix of the world's national incomes, of the countries expected incomes, and of the price of claims on these incomes. Under some restrictive assumptions, however, a much simpler strategy achieves the same allocation. The specific assumptions are: first, risk aversion is identical for young and old, second, all countries are following an optimizing policy, and third, macro markets exist. These markets allow old individuals to buy and sell shares in incomes. If these conditions are met, the government merely needs to transfer half of its share of world income from the old to the young in order to generate efficient risk sharing. There is no further need for governments to be concerned with world risk management. The international risk sharing is generated the macro markets. However, this result assumes that new financial markets are available that currently do not exist. Namely macro markets, or government bonds, indexed to the their own national income (see also Sect. 7.2.3).

We conduct our analysis of international diversification on a less ambitious scale. Instead of deriving optimal international risk sharing, we will take an empirical perspective and show how the analysis of efficient portfolio frontiers from Sect. 7.1.1 changes by including foreign assets in the basket of available investment possibilities. We do this in order to show that foreign assets provide a meaningful expansion to the portfolio for old-age provision. It is important to stress this point since in many countries, legislation constrains foreign investments by pension funds and life-insurers:

[20] Reisen (2000) also advocates international diversification of pension benefits. His argument is based mainly on exploiting the differences in macroeconomic savings between fast aging countries and slow aging countries; see also Sect. 3.2.

from a selection of seventeen OECD countries, Reisen (2000, p.18)) classifies five countries (Denmark, Finland, Germany, Sweden, and Norway) to have a tight level of restriction. Another five countries (Belgium, Canada, Japan, Portugal, and Switzerland) are classified as "medium" and the remaining seven countries (Australia, Ireland, Luxembourg, Netherlands, Spain, UK and USA) exhibit a loose level of restrictions.

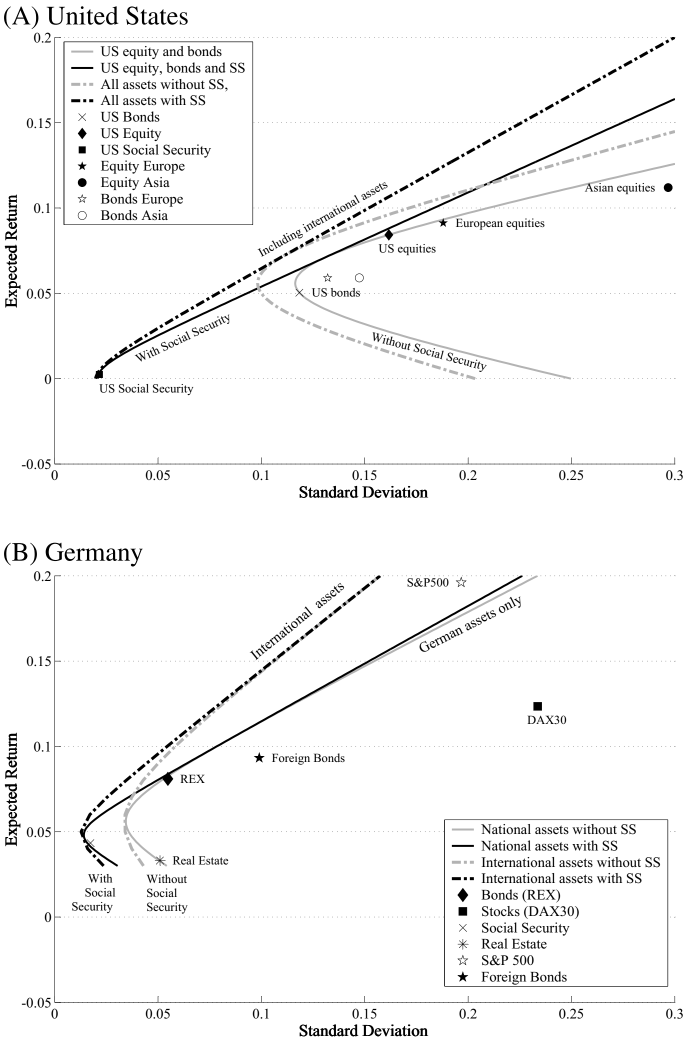
The efficiency frontiers for the case with international assets is depicted in Fig. 7.4. For the United States, the portfolio is expanded by equities and bonds from Europe and Asia, respectively. In the German case, we add the S&P500 and foreign bonds to the basket of assets.[21] The necessary statistics to compute the efficient portfolio frontier are given in Table 7.1. The portfolio compositions of the minimum variance portfolio and for a sample selection of returns (5%, 10%, 15%) are reported in Table B.4 in the Appendix.

In the German case, the inclusion of the S&P500 constitutes an investment possibility with a very high return that is otherwise not available. Thus investments in the S&P500 are beneficial for the portfolio of a German independent of its covariance. In the United States, the refinements of the portfolio are due more to risk diversification in the original sense. In both cases, portfolio risks can be reduced, especially if higher expected return are aimed for. The minimum variance portfolio is not so much affected by international diversification, because national social security remains the safest asset in both case studies. The international risk diversification, as Shiller (1999b) has it in mind, would also allow to include social security of other countries in the portfolio. While the data is not available for this scenario, it is not hard to imagine that this would constitute another expansion that is valuable for an improvement in the efficient portfolio frontier.

Some further remarks apply for the analysis of international diversification: we have looked at the efficient portfolio frontier on the basis of data on an annual frequency. For the issue of old-age provision, it would be interesting to consider the variance-covariance matrix for lower frequencies. Baxter and Jermann (1997) and Bohn (1999, Table V) have found a high correlation of capital and labor returns for long horizons. For wage-indexed social security, this implies high correlation of equity returns and social security for long horizons. The benefits of international investments should thus be even larger.

Also, we have not dwelled further on the importance of human capital as another major non-tradable asset of individuals during their working years, i.e. the accumulation phase. Baxter and King (2001) perform detailed calculation of the composition of human wealth and the correlation of returns from human capital with those of national and international financial assets. They conclude that their results are "consistent with there being important risk-management benefits to holding international assets since human capital returns are more highly correlated with domestic returns than with international returns"; Baxter and King (2001, p.428).

[21] We also include real estate for Germany; see above.



Source: Baxter and King (2001) and Author's calculation using data from Schacht (2001)

Fig. 7.4. Efficiency frontier with international assets and social security

7.1.4 Strategic Asset Allocation and Portfolio Choices Over the Life-Cycle

Until now, the analysis has been restricted to the utilization of the simple static mean-variance approach. The existence of human capital acting as an asset that is paying dividend in form of labor income and the notion that a long-term investor may evaluate risks differently has been recognized on a theoretical level for a long time.[22] Furthermore, some essential assumptions of the optimal short-term portfolio choice

[22] See discussion and literature cited in Campbell and Viceira (2002, p.6).

framework might be violated for longer horizons. Specifically, interest rates, the risk premium, and also the volatility of returns themselves may be changing over time.[23] Empirical approaches and applied work are, however, hardly available. This is due to the problem that for long-term optimal portfolio choice, closed-form solutions are only available for very special cases. Techniques for approximate solutions and computing power have, however, sparked new research in this field in the past years. In particular, Chaps. 6 and 7 in Campbell and Viceira (2002), Campbell et al. (2001), and Baxter and King (2001) integrate the existence of human wealth and life-cycle effects into the problem of long-term portfolio choice. In a sense, they are incorporating the roots of modern macroeconomics building on the work of Ando and Modigliani (1963) into the modern finance literature.[24] We confine ourselves here to a summary of the most important results derived in Campbell and Viceira (2002).

First, a long-term analysis treats bonds and money market investments very differently from the way that short-term analysis does. Money market investments are no longer risk-free, as they must be rolled over at an uncertain future interest rate. Instead, an inflation-indexed bond that is being held until maturity is a much safer investment. A conservative investor should thus hold long-term inflation-indexed bonds. For modest inflation risks, i.e. expected low future inflation rates, the same holds also for nominal long-term bonds.

Second, it is often asserted that stocks are much safer over long horizons than they appear from a shorter perspective.[25] In order for stocks to exhibit a reduction of risk at longer horizons, the excess-return of stocks must be mean-reverting. This is equivalent to a time-varying equity premium. Empirical evidence does point in this direction. However, this has the implication that instead of following a simple buy-and-hold strategy, the long-term investor should be an even more aggressive market timer than a myopic investor.

Third, human wealth plays an important role for the investor's true wealth. Labor income may be interpreted as a stream of dividends from this human wealth. The individual's human wealth is thus the expected present value of the sum of all future labor incomes. Human wealth has one very important distinction from other forms of wealth: it is non-tradable due to the obvious incentive restrictions.[26] As a first approach, one may take the income stream as deterministic, i.e. riskless, and therefore it resembles an investment into a riskless asset. The total wealth (W^T) thus equals financial wealth (W) and human wealth (H). If a tradable riskless asset exists, the implication for the portfolio is quite intuitive: before doing anything, the investor should short the risk-free asset at the amount of his human wealth (H) – at least in his head. Now he has W^T as tradable wealth at hand. The investor can then use the

[23] See Campbell and Viceira (2002), who tackle the problems separately in their Chaps. 3, 4, and 5, respectively.

[24] This synthesis has first been pioneered by Bodie et al. (1992).

[25] Glassman and Hasset (1999) actually claim that stocks are just as safe as bonds over a long holding period.

[26] This was the point of departure for Merton (1983) in arguing in favor of social security as a device of intergenerational risk sharing.

standard portfolio choice approach in order to derive the best equity-bond mix applied to total wealth, W^T. The percentage invested in equity measured in terms of financial wealth W, which we have denoted ω^p, should thus be larger under the existence of positive human wealth than without human wealth.[27] Hence, the financial portfolio should strongly tilt away from riskless assets towards risky assets. Because the human wealth decreases with aging, the equity share should be a decreasing function in the investor's age. Borrowing constraints may render the optimal portfolio infeasible and a corner solution at $\omega^p = 100\%$ may result in early ages. However, ω^p should still be a decreasing function in age (although not strictly decreasing). Taking account of the underlying risks of human wealth complicates the issue: if labor income is risky, especially if it is positively correlated with risky asset returns, the portfolio share in equity should be smaller than when human wealth is risk-free. In fact, if labor income has a large variance, the optimal ω^p is that of a retired person, i.e. time invariant. If labor income is highly variable and strongly positively correlated with risky asset returns, the optimal allocation may even be smaller under the presence of human wealth than without its presence. Further issues that emerge in such a setting are i) flexible labor supply may increase the willingness to take risks and ii) the saving-decisions, i.e. the total size of savings, may also depend on the stochastic properties of labor income.

Until now, the described results have been derived without taking explicit account of the life-cycle. However, an individual's life is composed of very specific phases: childhood and education, followed by working-life, and finally retirement. In addition, labor incomes usually take the form of a humped shaped profile over the working-life. Furthermore, retirement, i.e. the period without labor income, moves closer with every year passing. Since individuals are interested in smoothing their consumption-path, the savings pattern over the life-cycle will thus be composed of an accumulation phase and a decumulation phase. The full complexity of investments over the life-cycle can, so far, only be solved with numerical methods using further restrictive assumptions. Some basic results from this approach are (cf. Campbell and Viceira (2002, Chap. 7)): risky investments should be extremely attractive for younger (but not necessarily the youngest) households. Both the accumulation of tradable assets and the decline in human wealth lead to a lower inclination to hold equity later in life.[28] Heterogeneity matters more in a life-cycle approach than otherwise: the path of labor income and its correlation with other assets differs between household-groups. The incomes of self-employed college graduates are riskier and also closer correlated with stock return than is the case for other households. Therefore, this group should be more risk-averse in their portfolio choice, and should thus hold less equity than equally wealthy other households. Heterogeneity in risk aversion also has a larger impact in the life-cycle approach: for starters, higher risk aversion will lead to a lower equity-share by itself.

[27] More specifically: if the solution to the standard allocation problem is investing ω^p percent of financial wealth, W, into equity and $(1 - \omega^p)$ into bonds, than the new solution is investing $\omega^e W^T$ into equity and $(1 - \omega^e)W^T - H$ into bonds. The equity-share in financial wealth thus equals: $\omega^p = \omega^e \frac{W+H}{W}$, with $\omega^p > \omega^e$ for $W, H > 0$.

[28] Corner solutions may, however, prevent visible changes in real life.

As a secondary effect, risk-averse households have higher precautionary savings and therefore more financial wealth relative to human wealth. A higher ratio of financial wealth over human wealth will in turn, also lead to a lower desired level of equity-share in the portfolio.

To conclude, we address some of the caveats of the research summarized in the book of Campbell and Viceira (2002). In context of our specific perspective here, it has to be mentioned that neither international assets nor real estate are included in the analysis.[29] Also, in Chap. 7 of Campbell and Viceira (2002), social security is not analyzed in depth and is only considered as a forced-saving in a risk-free asset.[30] Generally, the frontier of this important strand of literature on *strategic asset allocation* is still on the level of applying partial equilibrium models to each single problem. According to Campbell and Viceira (2002, p.13), the reason for this is that "the academic literature has not yet developed a reliable, generally accepted model of the complete portfolio problem."

While this Chapter does not provide a practical implementation strategy for the retirement-saver, some findings have emerged that may serve as a rule-of-thumb.[31] Long term horizons, human wealth, the existence of social security, and the possibilities to diversify in a wide range of assets are all important factors that have to be considered for the interplay of social security and portfolio choice. This holds for both the individual investor (or the people consulting him) and the policy maker, who is designing social security and regulating the second and third pillar.

7.2 Social Security and Financial Instruments

In this section, we dwell on further issues where financial markets and social security are closely connected. In Sect. 7.2.1, we touch on the role of annuity products in the decumulation phase. In Sect. 7.2.2, we follow up on the issue of whether the equity premium can be exploited in order to solve the pension crises. Within such a funding strategy, minimum benefit guarantees represent latent costs that need to be considered. We conclude with referring to some innovative ideas for new financial products that authors have put forward to enhance risk sharing in the future.

[29] Baxter and King (2001) do include international diversification and model human wealth thoroughly in their analysis (see above).

[30] Campbell et al. (2001) compare model of life-cycle portfolio choice under systems where retirement wealth is invested 100/0 or 50/50 in riskless / risky assets. The 50/50 policy may increase welfare if some households do not participate in stock markets because of fixed costs.

[31] The reader who is looking for hands–on help for this sophisticated problem should visit www.ESPlanner.com and look at the financial planing software promoted there. Unfortunately, the software is only engineered for the U.S. saver.

7.2.1 Annuities and Inflation-Indexed Annuities

An annuity is a contract sold by an insurance company designed to provide payments to the holder at specified intervals. Because annuities are policies that usually pay the beneficiaries until the death of the beneficiary, these contracts provide protection against outliving ones resources. Thus, annuities are of special value for the liquidation of wealth during retirement because they can provide insurance against longevity risks. A wide variety of annuity products exist and not all contracts cover the same types of risks. We will use the term annuity referring to life annuities only.[32] Also, we will abstract from the accumulation phase and restrict the discussion to the liquidation phase. Annuities that begin with the liquidation right after purchase are called single-premium immediate annuities.

The product–variety of annuities is interesting because they enable individuals to replicate benefit payments from public pension schemes, but at the same time, provide others choices.[33] In the United States, the payout is either fixed or variable. Under the variable payout option, the individuals have the freedom to determine the investment strategy by choosing from a variety of funds. The annuity is then calculated using an assumed investment rate of return (AIR). The annuity will be adjusted periodically if the true return deviates from the AIR. In the U.K., which is probably the country with the most developed annuity markets, the fixed payout scheme can be enhanced by "graded" payments or inflation–indexed payments. The former type of annuity promises a nominal annual increase, typically 5 percent, over the life of the annuity product. The inflation-indexed annuity adjusts the annual payment to inflation. It is likely to assume that inflation-indexed annuities will only be offered by insurance companies in countries where inflation-indexed government debt is available. In the United Kingdom, inflation-indexed bonds (IL Gilt) have a long tradition with the first issue of the IL Gilt in 1981. Today, such bonds are available in a number of countries. Among them are the United States (TIPS), France (OATi), France for Euroland[34] (OAT€i), Canada (RRB), Sweden, Australia, New Zealand, and most recently, Italy.[35]

A stumbling fact in the analysis of annuity markets is that the demand is so low on a voluntary level. This is stumbling, because the life-cycle hypothesis predicts that an individual would want to annuitize a large portion of his wealth; cf. Yaari (1965). Several explanations have been put forward why consumers' interest

[32]Note that within this type of annuity, the options of "joint last survivor annuity" and "death benefits" usually exist.

[33]An in-depth analysis of the role of annuities in financing retirement can be found in Brown et al. (2001b).

[34]Meaning that the government of France has issued a bond that is indexed to the CPI of the Euroland.

[35]The global market capitalization of inflation-indexed bonds amounts to 350 billion € in 2002. Currently, the longest maturity of these securities is 30 years (U.S. TIPS and OAT€i). The coupons vary between 0 and 4.25 percent, the real yield for the 30 year bonds are slightly above 3 percent for the OAT€i and the U.S. TIPS.

in annuities is so low:[36] first, bequest motives or motives for transfers inter–vivos may prevent households from buying annuities on a large scale. Second, the possible need for discretionary spending during retirement, e.g. health care spending, makes non-annuitized wealth necessary. Third, the existence of a public pension program and company pensions crowds out the demand for annuity products. Finally, adverse selection raises the price of annuities and make them less attractive.

Adverse selection occurs because individuals with a limited life-expectation (due to some illness or medical condition) will not buy annuities. As a consequence, the mortality-table that insurers use to compute annuity prices take account of the higher survival rates of the group of people that purchase annuities. Poterba (2001a, Table 1) reports the mortality rates in the U.K. for the entire population in comparison to that rate for voluntary annuitants. The mortality rate of a 65 year old man is on average 2.12 percent. The same probability for a man who has voluntarily purchased an annuity is only 0.89 percent. Because insurance companies will need to take the adverse selection into account when setting their prices, an individual with average life-expectancy may therefore find purchasing an annuity a bad deal. How the prices of annuity products are affected by adverse selection can be measured by "moneysworth" calculations. The "moneysworth ratio" is defined as the ratio of the sum of the expected present value of the future payment stream (taking account of average survival probabilities) over the price of the annuity. Obviously, the actuarially fair moneysworth ratio is unity.[37]

Annuity prices in the United States were first analyzed by Friedman and Warshawsky (1988, 1990) and Warshawsky (1988). More recent studies can be found in Mitchell et al. (1997), Poterba (2001a), Mitchell (2002), and Brown et al. (2001b). Some stylized facts emerge: adverse selection increases annuity prices by 7–10 percent. The cost of adverse selection rises with the annuitants age. The costs are higher for male than for female. In addition, Poterba (2001a) finds that the price increase due to adverse selection is even more profound for inflation-indexed annuities and escalating annuities than for nominal fixed annuities. The existence of social security may also exacerbate the adverse selection problem. Since social security is a substitute for life annuities the demand for annuity is crowded out. Abel (1986) argues that the crowding out effect is disproportionately larger for individuals with shorter life-expectancy than for individuals with longer life-expectancy. Finally, the positive correlation between income and longevity induces adverse selection. Walliser (2000) uses a calibrated life-cycle model to simulate annuity prices. He finds that the elimination of social security would reduce the adverse selection-induced loading of annuity prices by 2–3 percentage points for 65 year old individuals. The simulations also show that roughly half of the measured adverse selection is due to the positive correlation of income and longevity. Walliser concludes that even within a scheme of

[36] See Poterba (2001a) and Walliser (2000).

[37] One may also find the calculation of a "load factor" in the literature. This factor measures in percent by how much the price of an annuity exceeds the actuarially fair price based on average mortality. The load factor is the inverse of the moneysworth ratio based on average mortality tables.

private accounts with compulsory annuitization, the adverse selection effect cannot be eliminated completely if private accounts are proportional to income.[38]

To conclude, we touch on the role of inflation-indexed annuities for privatized pension schemes. Brown et al. (2001a) cast doubt on the hypotheses that equity may provide "inflation insurance". In the past, stock returns have generally been very high on average, but returns do not move parallel to inflation. Inflation-indexed annuities thus seem to be a unique (private market) tool to hedge longevity and inflation risks simultaneously. Nevertheless, Brown et al. (2001a) put the benefit of this tool into perspective. Using a simulation model to estimate the retirees' willingness to pay for real, nominal, and variable equity-linked annuities, they find that inflation protection only has a modest value for plausible degrees of risk aversion. Instead, variable equity-linked annuities would be appreciated for their additional return.

7.2.2 Exploiting the Equity Premium versus Costs of Minimum Benefit Guarantees

A number of economists, most prominently Martin Feldstein, have argued that exploiting the equity premium can be a solution for the pension crises. Several authors have tried to verify this, others have tried to rebut this assertion: Abel (2001b) explores whether the trust fund can exploit the equity premium to generate welfare gains in a setting where some individuals are prevented from participating in the stock market due to fixed costs.[39]

Specifically, he considers a policy under which the social security trust fund is selling a dollar of bonds per capita and buying a dollar of equity per capita within a fully funded defined contribution scheme. He comes to the conclusion that it may be feasible to increase welfare for all living generations by such a policy in the constrained case, but that future generations are worse off, because this policy reduces savings and hence the aggregate capital stock. The reduction of savings occurs because the low income groups increase consumption in the young period in response to the positive income effect for this group. The positive income effect is generated by helping these groups participate in stock markets.

Feldstein et al. (2001) also address the question of whether the equity premium can be exploited in order to finance a transition from PAYG to a (partially) funded scheme. They simulate such a transition in the presence of a conditional intergenerational transfer from workers to retirees if old-age provisions fall short of the benchmark, i.e, the payments from the pure PAYG scheme. By conducting a large amount of simulations with constructed stochastic processes for returns, they come to the conclusion that in 2050, the "extra risk to taxpayers in providing this guarantee is very small"; Feldstein et al. (2001, p.80). MaCurdy and Shoven (2001) conduct a similar analysis, but their

[38] A mandatory annuitization further adds welfare-costs as individuals are loosing the freedom to choose according to their preferences (see Poterba (2001a)).

[39] For a similar analysis, see Diamond and Geanakoplos (1999) and Campbell et al. (2001). An expansion to imperfect annuity markets is found in Miles and Cerny (2001).

results are not as optimistic. Using historical data and bootstrap techniques, they estimate that 20 to 25 percent of the time, investing the trust fund in private securities worsens the financial situation of social security. To make things worse, failures are also autocorrelated, i.e. if the strategy fails in one year, it will most likely fail in the next year as well.

While the research of Feldstein et al. (2001) is certainly an important point of departure in the political debate to address the issues of aging society (see also Chap. 3.2), this approach has three shortcomings. First, the complete abolishment of PAYG social security eliminates the possibility of intergenerational risk sharing unless alternative securities are developed. Second, as it has been stressed by Abel (2001b), funding social security should lead to a reduction in the equity premium. Third, calculating the probability of welfare–reductions with the described "likelihood–method" may be misleading. Because loss–probabilities are not utility adjusted, the high aversion to infrequent but potentially large shocks is not taken into account. As an alternative, Smetters (1998, 2001) and Constantinides et al. (2002) resort to arbitrage pricing theory to put a price on the minimum benefit guarantee.[40] Using option pricing theory as developed by Black and Scholes (1973) and Merton (1973), Smetters (2001, p.95) finds that "arbitrage pricing suggests that the unfunded guarantee to replace social security benefits is quite costly. Replacing one unfunded pay-as-you-go benefit with a pay-when-needed guarantee can lead to only a small reduction in unfunded liabilities, and possibly even an increase." He further argues that replacing a fixed benefit scheme with a minimum guarantee of the same size is actually an increase in benefits, because higher returns are now possible for retirees but they do not bear the risk of the downside. Constantinides et al. (2002) refine the analysis by taking account of changes in the probability distribution due to equity investments by the trust fund. According to these authors, the value of the put-option resembles a temporary increase of the social security contribution rate of up to 25 percent. Still, their results are much less pessimistic towards pre-funding than those of Smetters. This is especially the case if the benefit guarantee is less than 100 percent.

The research surveyed in this section is closely related to the the study of demographic aging in the OLG framework; cf. Sect. 3.2.2. Both strands of literature address the question whether (pre)funding is a solution to the pension crises. The focus of the analysis here is the issue that funding is associated with a transferal of risk towards future generations, as they might have to act as the lender of last resort. Two approaches have been presented to quantify (or price) the risk of funding: the "likelihood method" and option pricing theory. While the results diverge across authors and methods considerably, a general notion can be found: partial funding with a minimum benefit guarantee less than 100 percent of the benchmark, i.e. the pure PAYG scheme, is most likely welfare improving without creating too much risk for future generations.

[40] A further alternative would be to use statistical measures of the downside risk, e.g. value at risk or lower partial moments; see Lahusen (2002). Bodie (2001) provides an exposition as to how arbitrage pricing theory can be used for financial engineering of retirement savings.

7.2.3 Potential New Financial Instruments: Innovative Government Debt Management and Macro-Markets

From the discussion of efficient portfolio choices in Sect. 7.1, we can see that it is hard to exploit risk diversification to its fullest extent because a number of state contingent markets do not exist. For example, the inclusion of social security improves the location of the efficient portfolio frontier in the mean-standard deviation diagram. It was nevertheless impossible for Germans to hold a share in the social security program of the United States (and vice versa). This possibility would be desirable in order to come closer to the kind of optimal international diversification Shiller (1999b) has described; cf. Sect. 7.1.3. Also, we have discussed that individuals cannot freely choose the size of their engagement in social security. Instead they are forced to invest a certain percentage of their income in social security. If the main argument in favor of having a social security scheme is risk diversification, one might argue that a laissez-faire approach, where an asset such as social security is provided but the portfolio weights are individually determined, should be preferred. The single investor can then decide by himself how his optimal portfolio, depending on his preferences over risk and return, and depending on his age, should look like. Unfortunately, such financial markets do not exist for the individual investor. Still, it is interesting to think how such markets could look like.

In his book from 1993, Robert Shiller pleas for the creation of macro markets, where securities are traded that let investors participate in a nation's GDP. With the existence of such markets in a number of countries, the biggest risks of national economies could be traded with other countries. In a similar line of argument, Bohn (2002) calls for GDP-indexed, wage-indexed and longevity-indexed government bonds as new tools for the allocation of aggregate risks. Bohn (2002) argues that especially wage-indexed bonds will be useful: both GDP-indexed and wage-indexed bonds help to share productivity risks, but GDP-indexed debt tends to amplify demographic shocks whereas wage-indexed debt does not. If factor prices respond to demographic changes, inflation-indexed bonds will actually share population risks. Longevity-indexed bonds could provide re-insurance for insurance firms selling annuities. Bohn comes to the conclusion that to reach perfect risk sharing, three elements are important: innovative public debt management, a wage-indexed defined-benefit public pension scheme, and capital income taxation.

In a very recent book, Shiller (2003), calls for a *New Financial Order*. He proposes six ideas that will increase the possibilities of risk allocation on both the individual level and on the aggregate level.[41] Shiller also shows that the creation of such markets are not solely the ideas of academics in an ivory tower. He cites a number of incidents where financial products have been linked to GDP: Citibank provided a loan for Bulgaria in 1994 under which interest rates where positively dependent on the Bulgarian GDP growth rate; Goldman Sachs and Deutsche Bank created the Economic

[41] In detail these are: i) insurance of livelihoods, ii) macro markets, iii) income-linked loans, iv) inequality insurance, v) intergenerational social security, and vi) international agreements for risk control.

Derivatives Market in 2002 where short-term options can be traded on macroeconomic variables (although not GDP); and also the IMF is thinking about fostering these products in less-developed countries.[42]

Whether these products will ever reach the stadium that their market capitalization and liquidity make them a serious vehicle for risk sharing for the means of old-age provision is to be questioned. The problem of asymmetric information associated with these products will probably grow with the size of the market for such products. However, if these markets can be set up in a functional way, risk may well be managed a lot more efficiently in the future than it is done today. After having analyzed issues of risk and risk sharing at length, one tends to be euphoric about these ideas. The implications of having institutions that provide risk sharing in a similar fashion as only social security is capable of doing today but within a market approach are compelling to an economist. However, we do raise some skepticism and conclude with *Another Viewpoint* by Harold Demsetz, who – in a discussion of Arrow's work on allocation of resources for invention – reminds us that it is a "fallacy of a free lunch" to "equate incomplete to nonoptimal. This would be correct only if commodity-options or other ways of adjusting risk are free." (Demsetz (1965, p.4)).

[42]See Shiller (2003, pp. 124–5, 142–5) for literature and further examples.

Chapter 8
Conclusion

In this book, we start our with an overview of the three main directions of arguments for the ongoing debate on social security. These are namely the fundamental results of social security under certainty, social security during demographic transition, and risk aspects of social security. The emphasis of this volume, however, lies on social security and issues of risk. Additionally, a specific focus on demographics is taken.

After an introduction to the basic foundations of social security, we touch on the issue of social security and demographic transition. From the literature covering macroeconomic effects during demographic transition, we conclude that the dispute on the influence of demographic transition on factor incomes and asset prices remains unsettled. Still, a general direction of results can be extracted: large generations are usually hit hardest by factor price movements during demographic transition. Thus, additional drastic reductions of the Baby-Boomers' pension benefits may result in a double burden for these generations. However, modelling the demographic transition in a closed economy framework tends to overestimate these effects as international capital flows will most likely attenuate factor income and asset price effects. Nevertheless, most authors find that the beneficial effects of international capital flows are at the margin and are not capable of substantially alleviating the burden of demographic transition for the fast aging countries.

The primary task here is the inclusion of risk aspects into the analysis of the ominous search for the holy grail of optimal social security. While we cannot offer *the* optimal solution, many important insights into what should be taken into consideration for this challenge are delivered, and some valuable rule-of-thumbs can be derived from them. First, risk sharing and risk diversification are important aspects in the quest for optimal old-age provision. Second, some of the largest macroeconomic risks can only be shared via government institutions such as social security.[1] Simply pointing to distributional effects of social security neglect this point. There are some merits to traditional social security, i.e. wage-indexed pay-as-you-go public pension schemes, when risk aspects are considered. Nevertheless, the Bismarckian principle of maintaining the level of living standards during retirement primarily via pay-as-you-go public pension programs has to be challenged. Forcing individuals to "invest" such a high share of wealth for old-age provision into social security has to be put in question; even against the background of risk sharing and risk diversification.

---

[1] Some theoretical restrictions need to apply for market failure, namely incomplete financial markets.

Third, old-age provision is subject to a number of macroeconomic risks. We have classified these as productivity and valuation risks, demographic risks, and political risks. A broad perspective of risk and risk sharing should be taken for designing social security. Furthermore, a subtle understanding of how these aggregate risks interact and how they are treated within the different pension systems is necessary. Fourth, it is a fallacy to view upon future benefits from public pension programs as “safe”. We show for a case study of the German benefit-rule that the political risks are of substantial magnitude.

Fifth, social security and other means of old-age provision should not be separated from each other. The optimal portfolio of an individual takes account of the presence of pay-as-you-go social security. At the same time, finding the optimal size of social security should incorporate a portfolio approach that takes the issue of risk diversification into consideration. Sixth, international risk diversification should not be hindered by government regulation. In many countries, regulations of investment possibilities for institutional vehicles for old-age provision, i.e. the second pillar and life-insurance, limit the potential benefits from international diversification. Finally, innovative ways of sharing these risks should be thought about. Among others, this includes thinking about a sensible policy concerning the design of public pensions.

In this context, a very specific idea is put forward in Chap. 5: pension benefits should be indexed to demographic developments such that demographic shocks are shared roughly equally between generations. The empirical results presented in Sect. 6 suggest that virtually demographic indexation is already effectively practiced. While there are some merits to having a pension scheme that is flexible enough to adjust to severe circumstances, one should nevertheless consider two advantages that would stem from amending the benefit rule explicitly by demographic indexation. One argument is political credibility. The other argument are the benefits of providing halfway reliable planning horizons for individuals.

In a sense, the introduction of optimal demographic indexation follows the concept of putting old-age provision on a wide base of risk exposure and thus risk sharing. This should be the guiding principle for old-age provision from a risk perspective for both the design of public pension programs and the diversification of assets in the other pillars.

Appendices

A Mathematical Appendix

A.1 Derivation of the Variance of y_t^{DC}

The variance of life-cycle resources under DC where the wage rate is dependent on cohort size can be calculated using y^{DC} and $\mathrm{E}[y^{DC}]$ from equations (5.17) and (5.18), respectively. By definition, the variance is then:

$$\begin{aligned}\mathrm{Var}[y_t^{DC}] &= \mathrm{E}\left[\left(y_t^{DC}\right)^2\right] - \left(\mathrm{E}[y_t^{DC}]\right)^2 \\ &= \mathrm{E}\left[\left(w(\varepsilon_t - \alpha\hat{\eta}_{t-1})(1-\gamma) + \frac{\gamma w}{1+r}(\varepsilon_{t+1} - \alpha\eta_t)(1+n+\eta_t)\right)^2\right] \\ &\quad - \left(w(1-\gamma) + \gamma w\frac{1+n}{1+r} - \alpha w\frac{\gamma}{1+r}\sigma_\eta^2\right)^2 .\end{aligned}$$

For multiplying the quadratic terms, one has to keep in mind that all random variables $(\varepsilon_t, \varepsilon_{t+1}, \eta_t, \hat{\eta}_{t-1})$ are assumed to be independent of each other. Among others the following moments are used: $\mathrm{E}[\varepsilon_{t+1}^2\eta_t^2] = (\sigma_\varepsilon^2 + 1)\sigma_\eta^2$, $\mathrm{E}[\eta_t^4] = 3\sigma_\eta^4$. Also note that all odd moments of η are zero. This is due to the assumption of η being normally distributed with mean zero. For all symmetric distributions, which applies for the normal, all odd central moments are zero. Since η is mean zero, all moments of η are equal to the respective central moments. After multiplying, employing the expectations operator and canceling terms we get the variance given in Eq. (5.19). Setting α equal to zero yields the variance of y^{DC} for the small open economy given in Eq. (5.8).

A.2 Derivation of the Variance of y_t^{DB} in a Small Open Economy

We first derive the variance of y^{DB} for the small open economy without macroeconomic feedback ($\alpha = 0$). From the definition of the variance we have:

$$\begin{aligned}\mathrm{Var}[y^{DB}] &= \mathrm{E}\left[\left(y_t^{DB}\right)^2\right] - \left(\mathrm{E}[y_t^{DB}]\right)^2 \\ &= \mathrm{E}\left[\left(w\varepsilon_t\left(1 - \frac{\psi}{1+n+\hat{\eta}_{t-1}}\right) + w\varepsilon_{t+1}\frac{\psi}{1+r}\right)^2\right] \\ &\quad - \left(w + \frac{w\psi}{1+r} - w\psi\,\mathrm{E}\left[\frac{1}{1+n+\hat{\eta}_{t-1}}\right]\right)^2 .\end{aligned}$$

Using the quadratic approximation for $(1+n+\eta)^{-1}$ given in Eq. (5.12) yields the quadratic approximation of the variance of y^{DB}:

$$\begin{aligned}\mathrm{Var}^{qa}[y^{DB}] =& w^2\,\mathrm{E}\left[\left\{\varepsilon_t\left(1-\psi\left(\frac{1}{1+n}-\frac{\hat{\eta}_{t-1}}{(1+n)^2}+\frac{\hat{\eta}_{t-1}^2}{(1+n)^3}\right)\right)\right.\right.\\ &\left.\left.+\,\varepsilon_{t+1}\frac{\psi}{1+r}\right\}^2\right]\\ &-w^2\left(\left(1-\frac{\psi}{1+n}\right)+\frac{\psi}{1+r}-\psi\left(\frac{1}{1+n}\right)^3\sigma_\eta^2\right)^2.\end{aligned} \tag{A.1}$$

After having substituted the quadratic approximation for $(1+n+\hat{\eta}_{t-1})^{-1}$, one only needs to take expectations over simple moments that can be derived by the moment generating function for the normal distribution. The quadratic approximation of the variance under DB then equals:

$$\begin{aligned}\mathrm{Var}[y^{DB}] =& w^2\left\{\left[\left(1-\frac{\psi}{1+n}\right)^2+\left(\frac{\psi}{1+r}\right)^2\right]\sigma_\varepsilon^2+\frac{\psi^2}{(1+n)^4}(1+\sigma_\varepsilon^2)\sigma_\eta^2\right.\\ &\left.-2\frac{\psi}{(1+n)^3}\left(1-\frac{\psi}{1+n}\right)\sigma_\eta^2\sigma_\varepsilon^2+\psi^2\left(\frac{1}{1+n}\right)^6(2\sigma_\eta^2+3\sigma_\eta^2\sigma_\varepsilon^2)\right\}.\end{aligned} \tag{A.2}$$

Deriving the linear approximation of this variance will be easier: when substituting for $(1+n+\hat{\eta}_{t-1})^{-1}$, the last term of the quadratic approximation can be neglected. The linear approximation is equal to the first line in Eq. (A.2).

A.3 Derivation of the Variance of y_t^{DB} with Macroeconomic Feedback

Substituting the quadratic approximation for $(1+n+\hat{\eta}_{t-1})^{-1}$ given in Eq. (5.12) into y_t^{DB} given in Eq. (5.17) yields the following variance:

$$
\begin{aligned}
\text{Var}[y^{DB}] =& w^2 \Bigg\{ \left[\left(1 - \frac{\psi}{1+n}\right)^2 + \left(\frac{\psi}{1+r}\right)^2 \right] (\sigma_\varepsilon^2 + \alpha^2 \sigma_\eta^2) \\
& + \frac{\psi^2}{(1+n)^4}(1+\sigma_\varepsilon^2)\sigma_\eta^2 \\
& + 2\frac{\psi^2}{(1+n)^4}\alpha^2\sigma_\eta^4 - 2\frac{\psi}{(1+n)^2}\left(1 - \frac{\psi}{1+n}\right)\alpha\sigma_\eta^2 \\
& - 2\frac{\psi}{(1+n)^3}\left(1 - \frac{\psi}{1+n}\right)\sigma_\eta^2\sigma_\varepsilon^2 + \frac{\psi^2}{(1+n)^6}(2\sigma_\eta^2 + 3\sigma_\eta^2\sigma_\varepsilon^2) \\
& - 6\frac{\psi}{(1+n)^3}\left(1 - \frac{\psi}{1+n}\right)\alpha^2\sigma_\eta^4 + 10\frac{\psi^2}{(1+n)^5}\alpha\sigma_\eta^2 \Bigg\}.
\end{aligned} \tag{A.3}
$$

A.4 Proof that $\mathrm{E}[y_t]$ is Strictly Increasing in ρ

Taking the partial derivative of the expectation of life-cycle resources given on the RHS in line one of Eq. (5.24) with respect to ρ yields:

$$
\frac{\partial\,\mathrm{E}[y_t]}{\partial \rho} = \gamma w \left((1+n)\,\mathrm{E}\left[\frac{1}{1+n+\hat{\eta}_{t-1}}\right] - 1 \right) \tag{A.4}
$$

Because $(1+n+\hat{\eta}_{t-1})^{-1}$ is strict convex for $\hat{\eta}_{t-1} < -(1+n)$, we have from Jensen's inequality that $\mathrm{E}\left[\frac{1}{1+n+\hat{\eta}_{t-1}}\right] > \frac{1}{1+n+\mathrm{E}[nt]}$ unless $\mathrm{E}[\hat{\eta}_{t-1}] = 0$ with probability one. This implies that:

$$
(1+n)\,\mathrm{E}\left[\frac{1}{1+n+\hat{\eta}_{t-1}}\right] > \frac{1+n}{1+n+\mathrm{E}[\hat{\eta}_{t-1}]} = 1. \tag{A.5}
$$

From equations A.4 and A.5 one can easily see that $\frac{\partial\,\mathrm{E}[y_t]}{\partial \rho} > 0$.

A.5 Derivation of the Variance Under the General Demographic Indexation Policy in a Closed Economy

Substituting the linear approximation of $(1+n+\hat{\eta}_{t-1})^{-1}$ into the second line of Eq. (5.26) yields:

$$
\begin{aligned}
y_t^{la} =& w(\varepsilon_t - \alpha\hat{\eta}_{t-1}) \left[1 - \gamma\left(1 - \frac{1-\rho}{1+n}\hat{\eta}_{t-1}\right) \right] \\
& + \gamma w(\varepsilon_{t+1} - \alpha\eta_t)\frac{1+n+\rho\eta_t}{1+r}
\end{aligned} \tag{A.6}
$$

The expectation of Eq. (A.6) is:

$$\mathrm{E}[y_t^{la}] = w(1-\gamma) + \gamma w \frac{1+n}{1+r} - \alpha\gamma w \left(\frac{1-\rho}{1+n} + \frac{\rho}{1+r}\right)\sigma_\eta^2,$$

so that the variance given in Eq. (5.28) can be derived by calculating:

$$\begin{aligned}\mathrm{Var}[y_t^{la}] = w^2 \Bigg\{ \mathrm{E}\Bigg[\Bigg((\varepsilon_t - \alpha\hat{\eta}_{t-1})\left[1-\gamma\left(1-\frac{1-\rho}{1+n}\hat{\eta}_{t-1}\right)\right] \\ + \gamma(\varepsilon_{t+1} - \alpha\eta_t)\frac{1+n+\rho\eta_t}{1+r}\Bigg)^2\Bigg] \\ - \left((1-\gamma) + \gamma\frac{1+n}{1+r} - \alpha\gamma\left(\frac{1-\rho}{1+n} + \frac{\rho}{1+r}\right)\sigma_\eta^2\right)^2 \Bigg\}.\end{aligned} \tag{A.7}$$

A.6 The Effect of Changing the Statistical Calculation of the Benefit Formula

The 20. RAG of 1977 postponed the retirement adjustment, and it changed the calculation of the dynamic factor AB. The lion's share of the generosity-cut is due to the new calculation of the AB. Because the calculation moved away from a three-year moving average towards an autoregressive determination, a relation of the growth rates of 1974, 1975, and 1976 is a determinant in all future calculations of AB. Because of the unusual high nominal growth rates of average earnings in these three years, this will lead to a reduction of generosity of the benefit formula if the average growth rate during pension payment is less than 8.3 percent. To derive this number, we use recursive substitution of the formulas for AB under the respective laws of 1957 and 1977 for a future year, say 2000:

$$AB_{2000}^{LAW57} = \frac{BE_{98} + BE_{97} + BE_{96}}{3} \tag{A.8}$$

$$\begin{aligned} AB_{2000}^{LAW77} &= \frac{BE_{75} + BE_{74} + BE_{73}}{BE_{76} + BE_{75} + BE_{74}} \cdot \frac{BE_{99} + BE_{98} + BE_{97}}{3} \\ &= \frac{1 + G_{74} + G_{74}G_{75}}{G_{74} + G_{74}G_{75} + G_{74}G_{75}G_{76}} \cdot \frac{BE_{99} + BE_{98} + BE_{97}}{3} \end{aligned}$$

Comparing the two leads to:

$$\frac{AB_{2000}^{LAW77}}{AB_{2000}^{LAW57}} = \frac{1 + G_{74} + G_{74}G_{75}}{G_{74} + G_{74}G_{75} + G_{74}G_{75}G_{76}} \cdot \frac{G_{97} + G_{97}G_{98} + G_{97}G_{98}G_{99}}{1 + G_{97} + G_{97}G_{98}}$$

or using the actual values for 74-76, $G_{74} = 1.114$, $G_{75} = G_{76} = 1.07$ and a constant growth rate during payout $G = 1 + g$:

$$\frac{AB_{2000}^{LAW77}}{AB_{2000}^{LAW57}} = 0.923 \cdot (1 + g) < 1 \text{ for } g < 0.0833$$

In comparison, the average growth rate since 1977 has been 3.5 percent with even much lower values in the past decade.

B Appendix: Tables

B.1 Regions and Country-Groups

Table B.1. Definition of regions / country-groups used in Chap. 3.1

Name	Countries
More developed countries	Europe, Northern America, Australia, New Zealand, and Japan
Less developed countries	Africa, Asia (excluding Japan), Latin America and the Caribbean plus Melanesia, Micronesia and Polynesia. Excluding all countries that are part of the least developed regions. This definition deviates from the one used by United Nations Population Division (2003a), i.e. the definition of less developed countries used here is identical to the UNDP's definition of "Less developed regions, excluding least developed countries".
Least developed countries	Afghanistan, Angola, Bangladesh, Benin, Bhutan, Burkina Faso, Burundi, Cambodia, Cape Verde, Central African Republic, Chad, Comoros, Democratic Republic of the Congo, Djibouti, Equatorial Guinea, Eritrea, Ethiopia, Gambia, Guinea, Guinea-Bissau, Haiti, Kiribati, Lao People's Democratic Republic, Lesotho, Liberia, Madagascar, Malawi, Maldives, Mali, Mauritania, Mozambique, Myanmar, Nepal, Niger, Rwanda, Samoa, São Tomé and Príncipe, Senegal, Sierra Leone, Solomon Islands, Somalia, Sudan, Togo, Tuvalu, Uganda, United Republic of Tanzania, Vanuatu, Yemen, and Zambia.
OECD	Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Japan, Korea, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Spain, Sweden, Switzerland, Turkey, United Kingdom, United States of America
G7	Canada, France, Germany, Italy, Japan, United Kingdom, United States of America

Source: United Nations Population Division (2003a)

B.2 Further Information on the GRV

Table B.2. Types of pensions and the respective adjustment factors

Type of pension	**adjustment factor (*RF*)**
Pension due to age (Rente wegen Alters)	1.0
Pension due to partially reduced capacity to work (Rente wegen teilweiser Erwerbsminderung)	0.5
Pension due to fully reduced capacity to work (Rente wegen voller Erwerbsminderung)	1.0
Survivor benefits for spouses under the age of 45 not raising children (limited to 2 years) (Rente wegen Todes: Kleine Witwen(r)renten)	0.25
Survivor benefits for spouses older than 45 or raising children (Rente wegen Todes: Große Witwen(r)renten)	0.55
Survivor benefits for children that have lost one parent (Rente wegen Todes: Halbwaisenrenten)	0.1
Survivor benefits for children that have lost both parent (Rente wegen Todes: Vollwaisenrenten)	0.2

Notes: Adjustment factors for pensions begining after 1.1.2001
Source: Sozialgesetzbuch VI, 2. Kap., 2. Ab., 3. Unterab.: Rentenhöhe und Rentenanpassung § 6

Table B.3. Determinants of the contribution rate (τ), least squares regression 1970–2001

	$\log(\tau_t)$				
	(1)	(2)	(3)	(4)	(5)
log(OAD[a])	0.12	0.07	0.14	0.20	0.17
	(0.82)	(0.45)	(0.92)	(1.27)	(1.02)
log(OAD(+17))	0.19**	0.15	-0.01	-0.06	-0.01
	(2.48)	(1.61)	(-0.09)	(-0.54)	(-0.13)
log(federal subsidy(+1))		0.09	0.07	0.03	0.09
		(1.02)	(0.79)	(0.37)	(0.99)
log(average earnings)			0.07	0.06	0.09*
			(1)	(0.92)	(1.86)
log(SSREV[b])				-0.02	
				(-1.12)	
dummy for unification					-0.02
					(-1.11)
AR(1)	0.49***	0.50**	0.41	0.30	0.31
	(2.88)	(2.75)	(1.4)	(0.97)	(0.98)
Constant	-1.40	-1.35	-2.18	-2.14	-2.37
Adjusted R^2	0.68	0.68	0.68	0.68	0.67
F-Statistic	22.97	17.46	13.93	11.95	11.68

t-statistics are in parentheses
significance is denoted by asterisks (*=10%, **=5%, ***=1%)

[a]Old-age dependency ratio: population 60+/ population(20–59).

[b]Reserves of the *GRV* measured in monthly expenditures (*Schwankungsreserve*).

B.3 Tables for Efficient Portfolio Frontier

Table B.4. Composition of efficient portfolios with international assets

Panel A: United States

		Portfolio Weight	Portfolio Weights Equities			Portfolio Weights Bonds		
Expect. Return	*Stand. Dev.*	Social Security	United States	Europe	Far East	United States	Europe	Asia
			Minimum Variance Portfolio					
-0.1	*2.0*	103.1	-5.5	3.6	-2.9	3.1	-2.1	0.8
			Efficient Portfolio for a Return of ...					
5	*7.9*	30.7	32.8	-9.1	6.7	6.7	31.0	1.1
10	*15.2*	-39.7	70.1	-21.3	16.1	10.2	63.3	1.4
15	*22.6*	-110.2	107.4	-33.6	25.4	13.7	95.5	1.7

Panel B: Germany

		Portfolio Weight	Portfolio Weights German Assets			Portfolio Weights Foreign Assets	
Expect. Return	*Stand. Dev.*	Social Security	Stocks (DAX)	Bonds (REX)	Real Estate	S&P 500	Bonds
			Minimum Variance Portfolio				
4.9	*1.3*	104.3	4.4	8.2	-8.7	2.3	-10.4
			Efficient Portfolio for a Return of ...				
5	*1.3*	103.7	4.3	9.1	-9.1	2.7	-10.8
10	*5.4*	68.8	2.8	65.3	-28.7	30.1	-38.3
0	*0.37*	33.9	1.3	121.4	-48.3	57.5	-65.7

Notes: All numbers in percent.
Source: Author's calculation using data from Baxter and King (2001) and Schacht (2001).

References

Aaron, H. J. (1966). The Social Insurance Paradox. *Canadian Journal of Economics and Political Science*, 33:371–374.

Abel, A. B. (1986). Capital Accumulation and Uncertain Lifetimes with Adverse Selection. *Econometrica*, 54:1079–1097.

Abel, A. B. (2001a). The Effects of Investing Social Security Funds in the Stock Market when Fixed Costs Prevent Some Houselholds from Holding Stocks. *American Economic Review*, 91(1):128–148.

Abel, A. B. (2001b). The Social Security Trust Fund, the Riskless Interest Rate, and Capital Accumulation. In Campbell, J. Y. and Feldstein, M., editors, *Risk Aspects of Investment-Based Social Security Reform*, pages 153–193. The University of Chicago Press, Chicago.

Abel, A. B. (2003). The Effects of Baby Boom on Stock Prices and Capital Accumulation in the Presence of Social Security. *Econometrica*, 71(2):551–578.

Abel, A. B., Mankiw, G. N., Summers, L. H., and Zeckhauser, R. J. (1989). Assessing Dynamic Efficiency: Theory and Evidence. *Review of Economic Studies*, 56:1–20.

Allen, F. and Gale, D. (1998). Financial Contagion. *Journal of Finance*, 53:1245–1284.

Ando, A. and Modigliani, F. (1963). The "Life Cycle" Hypothesis of Saving: Aggregate Implications and Tests. *American Economic Review*, 53:55–84.

Antolin, P. and Scarpetta, S. (1998). Microeconometric Analysis of the Retirement Decision: Germany. OECD Economics Department Working Papers No. 204, ECO/WKP(98)17.

Arnds, P. and Bonin, H. (2002). Frühverrentung in Deutschland: ökonomische Anreize und institutionelle Strukturen. IZA Discussion Paper No 666. Bonn.

Auerbach, A. J., Gokhale, J., and Kotlikoff, L. J. (1991). Generational Accounting: A Meaningful Alternative to Deficit Accounting. In Bradford, D., editor, *Tax Policy and the Economy*, volume 5, pages 55–110. MIT Press, Cambridge, MA.

Auerbach, A. J., Gokhale, J., and Kotlikoff, L. J. (1992). Generational Accounting: A New Approach to Understand the Effects of Fiscal Policy on Saving. *Scandinavian Journal of Economics*, 94:303–318.

Auerbach, A. J., Gokhale, J., and Kotlikoff, L. J. (1994). Generational Accounting: A Meaningful Way to Evaluate Fiscal Policy. *Journal of Economic Perspectives*, 8:73–94.

Auerbach, A. J. and Herrmann, H., editors (2002). *Ageing, Financial Markets and Monetary Policy*. Springer–Verlag, Berlin, Heidelberg.

Auerbach, A. J. and Kotlikoff, L. J. (1987). *Dynamic Fiscal Policy*. Cambridge University Press, Cambridge MA.

Banca d'Italia (2000). *Fiscal Sustainability*. Banca d'Italia.

Barbie, M., Hagedorn, M., and Kaul, A. (2000). Dynamic Efficiency and Pareto Optimality in a Stochastic OLG Model with Production and Social Security. IZA Discussion Paper, 209.

Barbie, M., Hagedorn, M., and Kaul, A. (2001). Government Debt as Insurance Against Macroeconomic Risk Is there a Role for Debt Beyond Curing Overaccumulation of Capital? IZA Discussion Paper, 412.

Barro, R. J. and Sala-i-Martin, X. (1995). *Economic Growth*. McGraw-Hill, New York.

Baxter, M. and Jermann, U. (1997). The International Diversification Puzzle is Worse Than You Think. *American Economic Review*, 87:170–180.

Baxter, M. and King, R. G. (2001). The Role of International Investment in a Privatized Social Security System. In Campbell, J. Y. and Feldstein, M., editors, *Risk Aspects of Investment-Based Social Security Reform*, pages 371–429. The University of Chicago Press, Chicago.

Bergantino, S. M. (1998). *Life Cycle Investment Behavior, Demographics, and Asset Prices*. PhD thesis, MIT, Cambridge, MA.

Besendorfer, D. (2003). *Sustainable Federalism – Theory and Application*. PhD thesis, Albert-Ludwigs-Universität, Freiburg.

Besendorfer, D., Borgmann, C., and Raffelhüschen, B. (1998). Ein Plädoyer für intergenerative Ausgewogenheit: Rentenreformvorschläge auf dem Prüfstand. *Ifo-Studien*, 44(3):209–231.

Birg, H. and Börsch-Supan, A. (1999). Für ein neue Aufgabenteilung zwischen gesetzlicher und privater Altersversorgung: Eine demographische und ökonomische Analyse. Gutachten für den Gesamtverband der deutschen Versicherungswirtschaft.

Black, F. and Scholes, M. S. (1973). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, 81(3):637–654.

Blanchard, O. (1985). Debt, Deficit, and Finite Horizons. *Journal of Political Economy*, 93(2):223–247.

Blöndal, S. and Scarpetta, S. (1998). The Retirement Decision in OECD Countries. OECD Economics Department Working Papers No. 202, ECO/WKP(98)15.

Bodie, Z. (2001). Financial Engineering and Social Security Reform. In Campbell, J. Y. and Feldstein, M., editors, *Risk Aspects of Investment-Based Social Security Reform*, pages 291–311. The University of Chicago Press, Chicago.

Bodie, Z., Marcus, A. J., and Meton, R. C. (1988). Defined Benefits and Defined Contribution Pension Plans: What are the Real Trade-offs? In Bodie, Z. and Shoven, J. B., editors, *Pensions in the U.S. Economy*, pages 139–160. The University of Chicago Press, Chicago.

Bodie, Z., Merton, R., and Samuelson, W. (1992). Labor Supply Flexibility and Portfolio Choice in a Life Cycle Model. *Journal of Economic Dynamics and Control*, pages 427–449.

Boeri, T., Börsch-Supan, A., and Tabellini, G. (2001a). Web Appendix for: Would You Like to Shrink the Welfare State? A Survey of European Citizens. http://www.economic-policy.org.

Boeri, T., Börsch-Supan, A., and Tabellini, G. (2001b). Would You Like to Shrink the Welfare State? A Survey of European Citizens. *Economic Policy*, 37:9–49.

Bohn, H. (1997). Social Security Reform and Financial Markets. In Sass, S. and Triest, R., editors, *Social Security Reform: Links to Savings, Investments, and Growth*, pages 193–227. Federal Reserve Bank of Boston, Boston, MA.

Bohn, H. (1998). Risk Sharing in a Stochastic Overlapping Generations Economy. Working Paper, University of California at Santa Babara,.

Bohn, H. (1999). Should the Social Security Trust Fund hold Equities? An Intergenerational Welfare Analysis. *Review of Economic Dynamics*, 2:666–697.

Bohn, H. (2001). Social Security and Demographic Uncertainty: The Risk-Sharing Properties of Alternative Policies. In Campbell, J. Y. and Feldstein, M., editors, *Risk Aspects of Investment-Based Social Security Reform*, pages 203–241. The University of Chicago Press, Chicago.

Bohn, H. (2002). Retirement Savings in an Aging Society: A Case for Innovative Government Debt Management. In Auerbach, A. J. and Herrmann, H., editors, *Ageing, Financial Markets and Monetary Policy*, pages 139–181. Springer–Verlag, Berlin, Heidelberg.

Bonin, H. (2001a). *Generational Accounting – Theory and Application*. Springer-Verlag, Berlin, Heidelberg.

Bonin, H. (2001b). Will it last? An Assessment of the 2001 German Pension Reform. *The Geneva papers on risk and insurance*, 26(4):547–564.

Borchert, E.-J. (1981). *Die Berücksichtigung familiärer Kindererziehung im Recht der gesetzlichen Rentenversicherung*. Duncker und Humblot, Berlin.

Borgmann, C. (2001). Assessing Social Security: Some Useful Results. Diskussionsbeiträge des Institut für Finanzwissenschaft der Albert-Ludwigs-Universität Freiburg, Freiburg, Nr 97/01.

Borgmann, C. (2002). Labor Income Risk, Demographic Risk, and the Design of (Wage-Indexed) Social Security. Diskussionsbeiträge des Institut für Finanzwissenschaft der Albert-Ludwigs-Universität Freiburg, Freiburg, Nr 100/02.

Borgmann, C. and Heidler, M. (2003). Demographics and Volatile Social Security Wealth: Political Risks of Benefit Rule Changes in Germany. CESifo Working Paper No. 1027, Munich.

Borgmann, C., Krimmer, P., and Raffelhüschen, B. (2001). Rentenreformen 1998 - 2001: Eine (vorläufige) Bestandsaufnahme. *Perspektiven der Wirtschaftspolitik*, 1:319–334.

Borgmann, C. and Raffelhüschen, B. (2004). *Die Entwicklung der Nachhaltigkeit der schweizerischen Fiskal- und Sozialpolitik: Generationenbilanzen 1995–2001*. Strukturberichterstattung Nr. 25. seco, Bern.

Breyer, F. and Straub, M. (1993). Welfare Effects of Unfunded Pension Systems When Labor is Endogenous. *Journal of Public Economics*, 50:77–91.

Brooks, R. (2000a). Population Ageing and Global Capital Flows in a Parallel Universe. IMF Working Paper WP/00/151.

Brooks, R. (2000b). What Will Happen to Financial Markets When the Baby Boomers Retire? IMF Working Paper WP/00/18.

Brown, J. R., Mitchell, O. S., and Poterba, J. M. (2001a). The Role of Real Annuities and Indexed Bonds in an Individual Accounts Retirement Program. In Campbell, J. Y. and Feldstein, M., editors, *Risk Aspects of Investment-Based Social Security Reform*, pages 321–360. The University of Chicago Press, Chicago.

Brown, J. R., Mitchell, O. S., Poterba, J. M., and Washawsky, M. (2001b). *The Role of Annuity Markets in Financing Retirement*. MIT Press, Cambridge, MA.

Browning, E. (1975). Why the Social Insurance Budget is too Large in a Democracy. *Economic Inquiry*, 13:373–388.

Börsch-Supan, A. (1992). Population Aging, Social Security Design, and Early Retirement. *Journal of Institutional and Theoretical Economics*, 148(4):533–557.

Börsch-Supan, A. (1999). Das Deutsche Rentenversicherungssystem. In *Gesetzliche Alterssicherung – Reformerfahrungen im Ausland: Ein systematischer Vergleich aus sechs Ländern*, pages 9–42. DIA – Deutsche Institut für Altersvorsorge, Köln.

Börsch-Supan, A. (2000a). Incentive Effects of Social Security on Labor Force Participation: Evidence in Germany and across Europe. *Journal of Public Economics*, 78:25–49.

Börsch-Supan, A. (2000b). Was lehrt uns die Empirie in Sachen Rentenreform. *Perspektiven der Wirtschaftspolitik*, 1(4):431–452.

Börsch-Supan, A., Heiss, F., Ludwig, A., and Winter, J. (2002a). Pension Reform, Capital Markets and the Rate of Return. *German Economic Review*, 4:151–181.

Börsch-Supan, A., Ludwig, A., and Winter, J. (2002b). Ageing and International Capital Flows. In Auerbach, A. J. and Herrmann, H., editors, *Ageing, Financial Markets and Monetary Policy*, pages 55–83. Springer–Verlag, Berlin, Heidelberg.

Bös, D. and Weizsäcker, R. (1989). Economic Consequences of an Aging Population. *European Economic Review*, 33:345–354.

Bütler, M. and Harms, P. (2001). Old Folks and Spoiled Brats: Why the Baby Boomers' Saving Crisis Need Not Be That Bad. Working Paper, Université de Lausanne.

Budd, A. and Campbell, N. (1998). The Role of the Public and Private Sectors in the U.K. Pension System. In Feldstein, M., editor, *Privatizing Social Security*, pages 99–134. The University of Chicago Press, Chicago.

Bundesministerium des Innern (1984). *Bericht über die Bevölkerungsentwicklung in der Bundesrepublik Deutschland 2. Teil: Auswirkungen auf die verschiedenen Bereiche von Staat und Gesellschaft.* Bundestags-Drucksache 10/863. Bundesministerium des Inneren, Bonn.

Campbell, J. Y. (2001). A Comment on James M. Poterba's "Demographic Structure and Asset Returns". *The Review of Economics and Statistics*, 83(4):585–588.

Campbell, J. Y., Cocco, J. a. F., Gomes, F. J., and Maenhout, P. J. (2001). Investing Retirement Wealth: A Life-Cycle Model. In Campbell, J. Y. and Feldstein, M., editors, *Risk Aspects of Investment-Based Social Security Reform*, pages 439–473. The University of Chicago Press, Chicago.

Campbell, J. Y. and Feldstein, M., editors (2001). *Risk Aspects of Investment-Based Social Security Reform*. The University of Chicago Press, Chicago.

Campbell, J. Y. and Viceira, L. M. (2002). *Strategic Asset Allocation*. Oxford University Press, Oxford.

Casamatta, G., Cremer, H., and Pestieau, P. (2000). The Political Economy of Social Security. *Scandinavian Journal of Economics*, 102(3):503–522.

Conde-Ruiz, I. J. and Profeta, P. (2002). What Social Security: Beveridgean or Bismarckian. Working Paper 633.

Constantinides, G. M., Donaldson, J. B., and Mehra, R. (2002). Junior Must Pay: Pricing the Implicit Put in Privatizing Social Security. NBER Working Paper No. 8906.

Coronado, J. L., Fullerton, D., and Glass, T. (2000). The Progressivity of Social Security. NBER Working Paper No. 7520.

Cutler, D. M., Poterba, J. M., Sheiner, L. M., and Summers, L. H. (1990). An Aging Society: Opportunity or Challenge? *Brooking Papers on Economic Activity*, 1:1–56.

Demange, G. (2002). On Optimality in Intergenerational Risk Sharing. *Economic Theory*, 20:1–27.

Demange, G. and Laroque, G. (1999). Social Security and Demographic Shocks. *Econometrica*, 67(3):527–542.

Demsetz, H. (1965). Information and Efficiency: Another Viewpoint. *The Journal of Law and Economics*, 12:1–22.

Diamond, P. (1965). National Debt in a Neoclassical Growth Model. *American Economic Review*, 55(5):1126–1150.

Diamond, P. (1997). Macroeconomic Aspects of Social Security Reform. *Brookings Papers on Economic Activity*, 2:1–87.

Diamond, P. (2002). An Assessment of the Proposals of the President's Commission to Strengthen Social Security. mimeo MIT.

Diamond, P. and Geanakoplos, J. (1999). Social Security Investment in Equities I: Linear Case. NBER Working Paper No 7103.

Disney, R. (1996). *Can We Afford to Grow Older?: A Perspective on the Economics of Aging*. MIT Press, Cambridge, Mass.

Eekhoff, J. (1985). Altersvorsorge bei sinkenden Bevölkerungszahlen. *Zeitschrift für Wirtschaftspolitik*, 34(3):247–269.

Eichner, T. and Wagener, A. (2004). Variance Vulnerability, Background Risks, and Comparative Statics with Mean-Variance Preferences. *The Geneva Papers on Risk and Insurance Theory*, forthcoming.

Eitenmüller, S. (1996). Die Rentabilität der gesetzlichen Rentenversicherung – Kapitalmarktanaloge Renditeberechnung für die nahe und ferne Zukunft. *Deutsche Rentenversicherung*, 12:784–798.

Enders, W. and Lapan, H. (1982). Social Security Taxation and Intergenerational Risk Sharing. *International Economic Review*, 23:647–658.

Fehr, H. (2000). Pension Reform during Demographic Transition. *Scandinavian Journal of Economics*, 102(3):419–443.

Feldstein, M. (1974). Social Security, Induced Retirement, and Aggregate Capital Accumulation. *Journal of Political Economy*, 82:905–926.

Feldstein, M. (1977). The Surprising Incidence of a Tax on Pure Rent: A New Answer to a Old Question. *Journal of Political Economy*, 85:349–360.

Feldstein, M. (1995). Fiscal Policy, Capital Formation and Capitalism. *European Economic Review*, 39:399–420.

Feldstein, M. (1998). *Privatizing Social Security*. The University of Chicago Press, Chicago.

Feldstein, M. and Horioka, C. (1980). Domestic Savings and International Capital Flows. *Economic Journal*, 90:314–329.

Feldstein, M. and Liebman, J. B. (2001). Social Security. NBER Working Paper 8451, Cambridge, MA.

Feldstein, M., Ranguelova, E., and Samwick, A. (2001). The Transition to Investment-Based Social Security When Portfolio Returns and Capital Profitability Are Uncertain. In Campbell, J. Y. and Feldstein, M., editors, *Risk Aspects of Investment-Based Social Security Reform*, pages 41–81. The University of Chicago Press, Chicago.

Feldstein, M. and Samwick, A. (1997). The Economics of Prefunding Social Security and Medicare Benefits. NBER Working Paper 6055, Cambridge, MA.

Fenge, R. (1995). Pareto-efficiency of the Pay-as-you-go Pension System with Intragenerational Fairness. *Finanzarchiv*, 52:357–363.

Fenge, R., Gebauer, A., Holzner, C., Meier, V., and Werding, M. (2003). *Alterssicherungssysteme im internationalen Vergleich: Finanzierung, Leistungen, Besteuerung*, volume 10 of *ifo Beiträge zur Wirtschaftsforschung*. ifo Institut für Wirtschaftsforschung, Munich.

Fetzer, S., Mevis, D., and Raffelhüschen, B. (2003). Zur Zukunfsfähigkeit des Gesundheitswesens. Eine Nachhaltigkeitsstudie zur marktorientierten Reform des deutschen Gesundheitssystems. Diskussionsbeiträge des Institut für Finanzwissenschaft der Albert-Ludwigs-Universität Freiburg, Freiburg, Nr 108/01.

Fisher, S. (1983). Welfare Aspects of Gonvernment Issue of Indexed Bonds. In Dornbusch, R. and Simonsen, M., editors, *Inflation, Debt and Indexation*, pages 223–246. MIT Press, Cambrdige MA.

Frankel, J. (1991). Quantifying International Capital Mobility in the 1980s. In Bernheim, B. and Shoven, J., editors, *National Saving and Economic Performance*, pages 227–269. University of Chicago Press, Chicago.

French, K. and Poterba, J. (1991). Investor Diversification and International Equity Markets. *American Economic Review, Papers and Proceedings*, 81(2):222–226.

Friedman, B. M. and J., W. M. (1990). The Cost of Annuities: Implications for Saving Behavior and Bequests. *Quaterly Journal of Economics*, 105(1):135–154.

Friedman, B. M. and J., W. M. (1998). Annuity Prices and Saving Behavior in the United States. In Bodie, Z., Shoven, J. B., and Wise, D., editors, *Pensions in the U.S. Economy*. University of Chicago Press, Chicaho.

Galasso, V. and Profeta, P. (2002). The Political Economy of Social Security: A Survey. *European Journal of Political Economy*, 18:1–29.

Gale, D. (1990). The Efficient Design of Public Debt. In Dornbusch, R. and Draghi, M., editors, *Public Debt Management: Theory and History*, pages 14–47. Cambridge University Press, Cambridge.

Geanakoplos, J., Magill, M., and Quinzii, M. (2002). Demography and the Long-Run Predictability of the Stock Market. Cowles Foundation Discussion Paper 1380.

Glassman, J. K. and Hasset, K. A. (1999). *Dow 36,000: The New Strategy for Profiting from the Coming Rise in the Stock Market*. Times Books, New York.

Gokhale, J. and Raffelhüschen, B. (1999). Population Aging and Fiscal Policy in Europe and the United States. *Economic Review*, 35(4):10–20.

Gordon, R. H. and Varian, H. R. (1988). Intergenerational Risk Sharing. *Journal of Public Economics*, 37:185–202.

Goyal, A. (2002). Demographics, Stock Market Flows, and Stock Returns. Anderson School of Management at UCLA, Mimeo.

Grohmann, H. (1980). *Rentenversicherung und Bevölerungsprognosen*. Campus-Verlag.

Gruber, J. and Wise, D. (1999). *Social Security and Retirement around the World*. The University of Chicago Press, Chicago.

Gruber, J. and Wise, D. (2002). Social Security Programs and Retirement Around the World: Micro Estimation. NBER Working Paper No. 9407.

Gustman, A. L. and Steinmeier, T. L. (2000). How Effective is Redistribution Under the Social Security Benefit Formula? NBER Working Paper No. 7597.

Hain, W., Eitenmüller, S., and Barth, S. (1997). Von Renditen, Gerechtigkeit und Reformvorschlägen. *Sozialer Fortschritt*, 46:210–226.

Hange, U. (2003). Public Pensions in an Overlapping Generations Model with Land: A Fairy Tale for Retirees. Mimeo, University of Munich.

Hansson, I. and Stuart, C. (1989). Social Security as Trade Among Living Generations. *American Economic Review*, 79(5):1182–1195.

Hassler, J. and Lindbeck, A. (1998). Intergenerational Risk Sharing, Stability and Optimality of Alternative Pension Systems. Institute for International Economic Studies, University of Stockholm.

Hauenschild, N. (1999). Uncertain Incomes, Pay-as-you-go Systems and Diversification. *Zeitschrift für Wirtschafts- und Sozialwissenschaften (ZWS)*, 119:491–507.

Höhn, C., editor (1998). *Demographische Trends, Bevölkerungswissenschaft und Politikberatung – Aus der Arbeit des Bundeinstituts für Bevölkerungsforschung (BiB), 1973 bis 1998*. Schriftenreihe des Bundesinstituts für Bevölkerungsforschung; Bd. 28. Leske + Budrich, Opladen.

Higgins, M. (1998). Demography, National Savings, and International Capital Flows. *International Economic Review*, 39(2):343–369.

Hirte, G. (2000). Privatizing Old-Age Insurance – Economic Effects and Political Feasibility. *Diskussionbeiträge der Katholischen Universität Eichstätt, Nr. 138.*

Holzmann, R. (2000). Can Investments in Emerging Markets Help to Solve the Aging Problem. Social Protection Discussion Paper Series No. 0010, World Bank.

Holzmann, R. and Stiglitz, J. E., editors (2001). *New Ideas about Old Age Security: Toward Sustainable Pension Systems in the 21st Century*. World Bank, Washington D.C.

Homburg, S. (1988). *Theorie der Alterssicherung*. Springer–Verlag, Berlin u.a.O.

Homburg, S. (1991). Interest and Growth in An Economy with Land. *Canadian Journal of Economics*, 24:450–459.

INGENUE (2000). Macroeconomic Consequences of Pension Reforms in Europe: An Investigation with the INGENUE World Model. CEPII Discussion Paper No. 17.

Jägers, T. and Raffelhüschen, B. (1999). Generational Accounting in Europe: An Overview. In *Generational Accounting in Europe*, number 6 in European Economy – Reports and Studies, pages 1–16.

Köhler, P. (1990). Entwicklungslinien der 100jährigen Geschichte der gesetzlichen Rentenversicherung: Die Rentenreform 1957. In Ruland, F., editor, *Handbuch der gesetzlichen Rentenversicherung*, pages 51–92. VDR.

Kohl, W. (1990). Die Entwicklung der Leistungen der gesetzlichen Rentenversicherung in den letzten 100 Jahren. *Deutsche Rentenversicherung*, 9–10:589–617.

Kotlikoff, L. and Raffelhüschen, B. (1999). Generational Accounting Around the Globe. *American Economic Review, Papers and Proceedings*, 89(2):161–166.

Kotlikoff, L. J., Smetters, K., and Walliser, J. (1999). Privatizing Social Security in the United States: Comparing the Options. *Review of Economic Dynamics*, 2:532–574.

Krimmer, P. and Raffelhüschen, B. (2003). Intergenerative Umverteilung und Wachstumsimpulse der Steuerreformen 1999 bis 2005: Die Perspektive der Generationenbilanz. Diskussionsbeiträge, Institut für Finanzwissenschaft der Albert-Ludwigs-Universität Freiburg, Nr. 105.

Krueger, D. and Kubler, F. (2002). Intergenerational Risk-Sharing via Social Security when Financial Markets Are Incomplete. *American Economic Review, Papers and Proceedings*, 92(2):407–410.

Lahusen, R. (2002). *Asset Allocation für die Alterssicherung*. Deutscher Universitäts-Verlag, Wiesbaden.

Lang, G. (1996). *On Overlapping Generations Models with Productive Capital*. Lecture Notes in Economics and Mathematical Systems. Springer, Berlin, Heidelberg.

Lee, R. D. and Tuljapurkar, S. (2001). Uncertainty and the Design of Long–Run Fiscal Policy. In Auerbach, A. J. and Lee, R. D., editors, *Demographic Change and Fiscal Policy*, pages 7–57. Cambridge Univerity Press, Cambridge, U.K.

Leslie, P. (1945). On the Use of Matrices in Certain Population Mathematics. *Biometrica*, 33:183–212.

Lim, K.-M. and Weil, D. N. (2003). The Baby Boom and the Stock Market Boom. Brown University, Department of Economics, Working Paper 2003-07.

Lindbeck, A. and Persson, M. (2002). The Gains from Pension Reform. Institute for International Economic Studies, Stockholm University, Paper No. 712.

MaCurdy, T. E. and Shoven, J. B. (2001). Asset Allocation and Risk Allocation: Can Social Security Improve Its Future Solvency Problem by Investing in Private Securities? In Campbell, J. Y. and Feldstein, M., editors, *Risk Aspects of Investment-Based Social Security Reform*, pages 11–32. The University of Chicago Press, Chicago.

Mankiw, G. M. and Weil, D. N. (1989). The Baby Boom, the Baby Bust and the Housing Market. *Regional Science and Urban Economics*, 19:235–258.

Mankiw, N. G. and Zeldes, S. P. (1991). The Consumption of Stockholders and Nonstockholders. *Journal of Financial Economics*, 29(1):97–112.

Markowitz, H. (1959). *Portfolio Selection: Efficient Diversification of Investments*. Wiley, New York.

Matsen, E. and Thøgersen, O. (2004). Designing Social Security – A Portfolio Choice Approach. *Journal of the European Economic Association*, forthcoming.

McHale, J. (2001). The Risk of Social Security Benefit-Rule Changes: Some International Evidence. In Campbell, J. Y. and Feldstein, M., editors, *Risk Aspects of Investment-Based Social Security Reform*, pages 247–282. The University of Chicago Press, Chicago.

Merton, R. C. (1972). An Analytical Derivation of the Efficient Portfolio Frontier. *Journal of Financial and Quantitative Analysis*, 7:1851–1872.

Merton, R. C. (1973). Theory of Rational Option Pricing. *Bell Journal of Economics*, 4(1):141–183.

Merton, R. C. (1983). On the Role of Social Security as a Means for Efficient Risk Sharing in an Economy where Human Capital is Not Tradeable. In Bodie, Z. and Shoven, J. B., editors, *Financial Aspects of the United States Pension System*, pages 325–358. University of Chicago Press, Chicago.

Miles, D. (1999). Modelling the Impact of Demographic Change upon the Economy. *The Economic Journal*, 109:1–36.

Miles, D. and Cerny, A. (2001). Risk, Return and Portfolio Allocation under Alternative Pension Arrangements with Imperfect Financial Markets. CESifo Working Paper No. 441, Munich.

Mitchell, O. (2002). Developments in Decumulation: The Role of Annuity Products in Financing Retirement. In Auerbach, A. J. and Herrmann, H., editors, *Ageing, Financial Markets and Monetary Policy*, pages 97–125. Springer–Verlag, Berlin, Heidelberg.

Mitchell, O., Poterba, J., and Warshawsky, M. J. (1997). New Evidence on the Money's Worth of Individual Annuities. NBER Working Paper No 6002, Cambridge MA.

Munnell, A. H. (1974). The Impact of Social Security on Personal Savings. *National Tax Journal*, 29:553–567.

OECD (2000). *Reforms for an Ageing Society*. Social Isuues. Organisation for Economic Cooperation and Development, Paris.

OECD (2001). *Ageing and Income: Financial Resources and Retirement in 9 OECD Countries*. Organisation for Economic Cooperation and Development, Paris.

Pestieau, P. and Possen, U. M. (2000). Investing Social Security in the Equity Market. Does it Make a Difference? *National Tax Journal*, 53(1):41–57.

Poterba, J. (2001a). Annuity Markets and Retirement Security. Center for Retirement Research at Boston College, Working Paper 2001-10.

Poterba, J. (2001b). Demographic Structure and Asset Returns. *Review of Economics and Statistics*, 83(4):565–584.

Poterba, J., Rauh, J., Venti, S., and Wise, D. (2003). Utility Evaluation of Risk in Retirement Saving Accounts. NBER Working Paper 9892.

Poterba, J., Venti, S., and Wise, D. (2000). Saver Behavior and 401(k) Retirement Wealth. *American Economic Review, Papers and Proceedings*, 90(2):297–302.

Raffelhüschen, B. (1989a). Alterssicherung und Staatsverschuldung. *Finanzarchiv*, 47:60–76.

Raffelhüschen, B. (1989b). *Anreizwirkungen der sozialen Alterssicherung: Eine dynamische Simulationsanalyse*, volume 40 of *Finanzwissenschaftliche Schriften*. Verlag Peter Lang, Frankfurt a.M.

Raffelhüschen, B. (1993). Funding Social Security Through Pareto-optimal Conversion Policies. *Journal of Economics*, 7:105–131.

Raffelhüschen, B. (1999a). Generational Accounting in Europe. *American Economic Review, Papers and Proceedings*, 89(2):167–170.

Raffelhüschen, B. (1999b). *Generational Accounting: Method, Data and Limitations*, volume 6 of *Reports and Stuides*, chapter 2, pages 17–28. European Commission, Brussels.

Raffelhüschen, B. (2002). Generational Accounting - Quo Vadis? *Nordic Journal of Political Economy*, 28:75–89.

Raffelhüschen, B. and Borgmann, C. (2001). *Zur Nachhaltigkeit der schweizerischen Fiskal- und Sozialpolitik: Eine Generationenbilanz*, volume 3 of *Strukturberichterstattung*. seco, Bern.

Raffelhüschen, B. and Risa, A. E. (1997). Generational Accounting and Intergenerational Welfare. *Public Choice*, 93:149–163.

Rangel, A. and Zeckhauser, R. (2001). Can Market and Voting Institutions Generate Optimal Intergenerational Risk Sharing? In Campbell, J. Y. and Feldstein, M., editors, *Risk Aspects of Investment-Based Social Security Reform*, pages 113–141. The University of Chicago Press, Chicago.

Razin, A., Sadka, E., and Swagel, P. (2002). The Aging Population and the Size of the Welfare State. *Journal of Political Economy*, 110:900–918.

Reisen, H. (2000). *Pensions, Savings and Capital Flows: From Ageing to Emerging Countries*. Elgar, Cheltenham.

Richter, W. F. (1993). Intergenerational Risk Sharing and Social Security in an Economy with Land. In Felderer, B., editor, *Public Pension Economics, Journal of Economics*, pages 91–103. Suppl. 7.

Ríos-Rull, J.-V. (2001). Population Changes and Capital Accumulation: The Aging of the Baby Boom. Working Paper University of Pennsylvania, Department of Economics.

Ruland, F. (1989). Das Rentenrecht – Neuregelungen durch das Rentenreformgesetz 1992. *Deutsche Rentenversicherung*, 12:741–782.

Ruland, F., editor (1990). *Handbuch der gesetzlichen Rentenversicherung – Festschrift aus Anlass des 100jährigen Bestehens der gesetzlichen Rentenversicherung*. VDR – Verband Deutscher Rentenversicherer, Frankfurt.

Saas, S. A. and Triest, R. K., editors (1997). *Social Security Reform: Links to Saving, Investment, and Growth*, volume 41 of *Conference Series*. Federal Reserve Bank of Boston, Boston.

Samuelson, P. A. (1958). An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money. *Journal of Political Economy*, 66(6):467–482.

Schacht, U. (2001). *Finanztheoretische Analyse der Altersicherung*. Europäische Hochschulschriften. Peter Lang, Frankfurt am Main.

Schnabel, R. (1998). Rates of Return of the German Pay-As-You-Go Pension System. *Finanzarchiv*, 55:374–399.

Schwarze, R., Deutsch, M., Krysiak, D., and Stewart, L. H. (2003). Intergenerational Justice and Sustainability – Economic Theory and Measurement. Background paper for an expert workshop at the DIW Berlin.

Shiller, R. J. (1993). *Macro Markets: Creating Institutions for Managing Society's Largest Economic Risks*. Clarendon Press, Oxford.

Shiller, R. J. (1999a). Human Behavior and the Efficiency of the Financial System. In Taylor, J. and Woodford, M., editors, *Handbook of Macroeconomics*, volume 1c, chapter 20. North Holland, Amsterdam.

Shiller, R. J. (1999b). Social Security and Institutions for Intergenerational, Intragenerational and International Risk Sharing. *Carnegie-Rochester Series in Public Policy*, 50:165–204.

Shiller, R. J. (2003). From Efficient Markets Theory to Behavioral Finance. *Journal of Economic Perspectives*, 17(1):83–104.

Sinn, H.-W. (1995). A Theory of the Welfare State. *Scandinavian Journal of Economics*, 97:495–526.

Sinn, H.-W. (1998). The Pay-As-You-Go Pension System as a Fertility Insurance and Enforcement Device. NBER Working Paper 6610, Cambridge, MA.

Sinn, H.-W. (2000). Pension Reform and Demographic Crisis. Why a Funded System is Needed and Why it is Not Needed. *International Tax and Public Finance*, 7:389–410.

Sinn, H.-W. and Thum, M. (1999). Gesetzliche Rentenversicherung: Prognosen im Vergleich. *Finanzarchiv*, 56:104–135.

Smetters, K. (1998). Privatizing Versus Prefunding Social Security. Working Paper, University of Pennsylvania.

Smetters, K. (2000). The Equivalence of the Social Security's Trust Fund Portfolio Allocation and Capital Inome Tax Policy. Working Paper, University of Pennsylvania.

Smetters, K. (2001). The Effect of Pay-When-Needed Benefit Guarantees on the Impact of Social Security Privatization. In Campbell, J. Y. and Feldstein, M., editors, *Risk Aspects of Investment-Based Social Security Reform*, pages 91–104. The University of Chicago Press, Chicago.

Smith, A. (1982). Intergenerational Transfers as Social Insurance. *Journal of Public Economics*, 19:97–106.

Sommer, B. (1992). Entwicklung der Bevölkerung bis 2030: Ergebnis der siebten koordinierte Bevölkerungsvorausberechnung. *Wirtschaft und Statistik*, 4:217–222.

Sommer, B. (1994). Entwicklung der Bevölkerung bis 2040: Ergebnis der achten koordinierte Bevölkerungsvorausberechnung. *Wirtschaft und Statistik*, 7:497–503.

Sommer, B. (2001). Entwicklung der Bevölkerung bis 2050: Ergebnisse der 9. koordinierten Bevölkerungsvorausberechnung des Bundes und der Länder. *Wirtschaft und Statistik*, 1:22–29.

SSA (1999). Social Security Programs Throughout the World, 1999. Social Security Administration.

SSA (2002a). Social Security Programs Throughout the World: Asia and the Pacific. Social Security Administration.

SSA (2002b). Social Security Programs Throughout the World: Europe. Social Security Administration.

Statistisches Bundesamt (1971). *Bericht über die Bevölkerungsentwicklung in der Bundesrepublik Deutschland 2. Teil: Auswirkungen auf die verschiedenen Bereiche von Staat und Gesellschaft.* Fachserie A Bevölkerung und Kultur, Reihe 1 Bevölkerungsstand und Entwicklung. W. Kohlhammer, Stuttgart und Mainz.

Steffen, J. (2002a). Die Anpassung der Renten in der Rentenversicherung der Arbeiter und Angestellten – Von der Rentenreform 1957 bis zum AVmEG. Mimeo, http://www.arbeitnehmerkammer.de/sozialpolitik.

Steffen, J. (2002b). Sozialpolitische Chronik: Die wesentlichen Änderungen in der Arbeitslosen-, Renten-, Kranken- und Pflegeversicherung, sowie bei der Sozialhilfe (HLU) - von den siebzigern bis heute. http://www.arbeitnehmerkammer.de/sozialpolitik.

Stiglitz, J. E., Munnell, A., and Frankel, J. (1997). *Economic Report of the President*. United States Government Printing Office, Washington, DC.

Stock, J. H. and Wise, D. A. (1990a). Pensions, the Option Value of Work, and Retirement. *Econometrica*, 58:1151–1180.

Stock, J. H. and Wise, D. A. (1990b). The Pension Inducement to Retire: An Option Value Analysis. In Wise, D. A., editor, *Issues in the Economics of Aging*, pages 205–230. The University of Chicago Press, Chicago.

SVR – Sachverständigenrat zur Begutachtung der gesamtwirtschaftlichen Lage (2001). *Für Stetigkeit – Gegen Aktionismus*. Metzler-Poeschel, Stuttgart.

SVR – Sachverständigenrat zur Begutachtung der gesamtwirtschaftlichen Lage (2002). *Zwanzig Punkte für Beschäftigung und Wachstum*. Metzler-Poeschel, Stuttgart.

Taylor, A. (1996). International Capital Mobility in History: The Saving-Investment Relationship. *NBER Working Paper*, 5743.

Thøgersen, O. (1998). A Note on Intergenerational Risk Sharing and the Design of Pay-As-You-Go Pension Programs. *Journal of Population Economics*, 11:373–378.

Thøgersen, O. (2003). The Risk Sharing Effects of Social Security and the Stochastic Properties of Income Growth. NHH Discussion Paper 07/03.

Tirole, J. (1985). Asset Bubbles and Overlapping Generations. *Econometrica*, 53(6):1499–1528.

Tobin, J. (1958). Liquidity Preferences as Behavior towards Risk. *Review of Economic Studies*, 25:68–85.

Turner, D., Giorno, C., De Serres, A., Vourc'h, A., and Richardson, P. (1998). The Macroeconomic Implications of Ageing in a Global Context. OECD Economics Department Working Paper Bo. 193.

United Nations Population Division (2003a). *World Population Prospects: The 2002 Revision*. United Nations, New York.

United Nations Population Division (2003b). *World Population Prospects: The 2002 Revision. The Highlights*. United Nations, New York.

Valdś-Prieto, S. (2000). The Financial Stability of Notional Account Pensions. *Scandinavian Journal of Economics*, 102(3):396–417.

VDR (2001). *Rentenbestand am 31. Dezember 2000*. VDR Statistik, Band 136. Verband Deutscher Rentenversicherungsträger, Frankfurt.

VDR (2002a). *Rentenversicherung in Zeitreihen 2002*. DRV-Schriften, Band 22. Verband Deutscher Rentenversicherungsträger, Frankfurt.

VDR (2002b). *Rentenzugang des Jahres 2001*. VDR Statistik, Band 141. Verband Deutscher Rentenversicherungsträger, Frankfurt.

Wagener, A. (2003a). On Intergenerational Risk Sharing within Social Security Schemes. *European Journal of Political Economy*, forthcoming.

Wagener, A. (2003b). Pensions as a Portfolio Problem: Fixed Contribution Rates vs. Fixed Replacement Rates Reconsidered. *Journal of Population Economics*, 16:111–134.

Walliser, J. (1997). Essays on Annuity Pricing and Social Security Privatization. Mimeo, Boston University.

Walliser, J. (2000). Adverse Selection in the Annuities Market and the Impact of Privatizing Social Security. *The Scandinavian journal of economics*, 102:373–393.

Warshawsky, M. J. (1988). Private Annuity Markets in the US. *Journal of Risk and Insurance*, 55(3):518–529.

Weiss, L. (1979). The Effects of Money Supply on Economic Welfare in the Steady State. *Econometrica*, 48:565–576.

Welch, F. (1979). Effects of Cohort Size on Earnings: the Baby Boom Babies' Financial Bust. *Journal of Political Economy*, 87:65–97.

Werding, M. (2000). Rentenreform: Modellrechnungen zu den langfristigen Effekten. *Ifo-Schnelldienst*, 53(28/20):39–42.

Werding, M. (2003). After Another Decade of Reform: Do Pension Systems in Europe Converge? *CESifo DICE Report*, 1:11–16.

World Bank (1994). *Averting the Old Age Crisis: Policies to Protect the Old and Promote Growth*. A World Bank Policy Research Report. Oxford University Press, Oxford.

Yaari, M. E. (1965). Uncertain Lifetime, Life Insurance, and the Theory of the Consumer. *Review of Economic Studies*, 32:137–150.

Yoo, P. S. (1994). Age distribution and Returns of Financial Wealth. Working Paper of the Federal Reserve Bank of St. Louis No 94-003A.

Young, A. (2001). Demographic Fluctuations, Generational Welfare and Intergenerational Transfers. NBER Working Paper 8530, Cambridge, MA.

Zweifel, P., Felder, S., and Meiers (1999). Ageing of Population and Health Care Expenditure: A Red Herring. *Health Economics*, 8:485–496.